AP CALCULUS AB REVIEW FOR UNIT 6 EXAM

(Questions in BOLD allow calculator)

1. $\int \left(\sqrt[3]{x}\right)^2\, dx$ $= \int x^{\frac{2}{3}}\, dx = \frac{3x^{\frac{5}{3}}}{5} + C$

2. $\int (4x-9)^{15}\, dx$ $= \frac{1}{4}\int u^{15}\, du = \frac{1}{4} \cdot \frac{u^{16}}{16} + C$

$u = 4x-9$

$du = 4\, dx$

$\frac{1}{4}du = dx$

$= \frac{1}{64}(4x-9)^{16} + C$

3. $\int 3y^2 \frac{dy}{\sqrt{y^3-4}}$ $= \int (y^3-4)^{-\frac{1}{2}}\, 3y^2\, dy = \int u^{-\frac{1}{2}}\, du = 2u^{\frac{1}{2}} + C$

$u = y^3-4$

$du = 3y^2\, dy$

$= 2\sqrt{y^3-4} + C$

4. $\int (3x+2)^8\, dx$ $= \frac{1}{3}\int u^8\, du = \frac{1}{3} \cdot \frac{u^9}{9} + C$

$u = 3x+2$

$du = 3\, dx$

$\frac{1}{3}du = dx$

$= \frac{1}{27}(3x+2)^9 + C$

5. $\int \frac{dx}{x+3} =$ $= \int \frac{1}{u}\, du$

$u = x+3$

$du = dx$

$= \ln|u| + C$

$= \ln|x+3| + C$

6. **Scientists begin to monitor a moose population in the year 2005. In 2010, there are 128 moose, and in 2015, there are 190 moose. If the population follows an exponential growth model, how many moose will there be in 2020?**

$y = y_0 e^{kt}$

$y_0 = 128$

$y(5) = 190$

Find $y(10)$

$190 = 128e^{5k}$

$k = .07899...$

$y = 128e^{(.07899...)(10)}$

$y = 282.031 \approx 282$ moose

7. $\int x^2 \cos x \, dx$

$u = x$ $\quad dv = \cos x \, dx$

$du = dx$ $\quad v = \sin x$

$uv - \int v \, du = x(\sin x) - \int \sin x \, dx$

$= x(\sin x) + \cos x + C$

8. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

$(0,0) = 0$

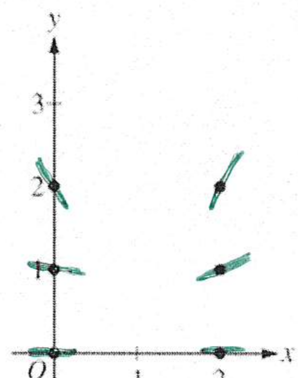
$(0,1) = -1$

$(0,2) = -4$

$(2,0) = 0$

$(2,1) = 1$

$(2,2) = 4$



Tangent Line:

$y - 3 = 9(x-2)$

$y = 9x - 15$

$y(2.1) = 3.9$

$x = 2$

$y = 3$

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $\boxed{f(2) = 3}$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$. $m = 9$ pt $(2, 3)$

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

$\frac{dy}{dx} = \frac{y^2}{x-1}$

$y^{-2} \, dy = \frac{1}{x-1} \, dx$

$u = x - 1$

$du = dx$

$\frac{y^{-1}}{-1} = \ln|x-1| + C$

$\frac{-1}{y} = \ln|x-1| + C$

$-\frac{1}{3} = C$

$\frac{-1}{y} = \ln|x-1| - \frac{1}{3}$

$\frac{-1}{y} = \frac{3\ln|x-1| - 1}{3}$

$y = \frac{-3}{3\ln|x-1| - 1}$

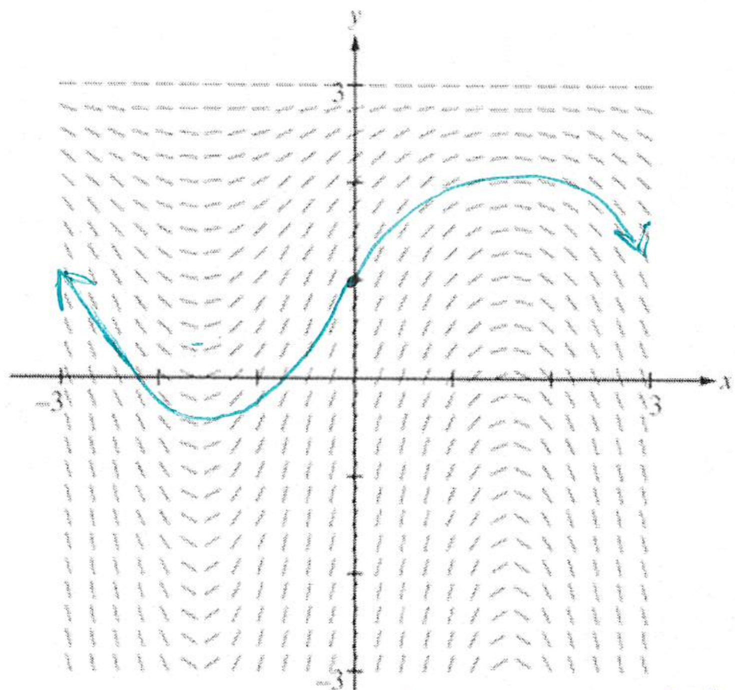
9. **OLD STUFF: Limits, Derivative Rules, Implicit Differentiation, Derivatives of Inverse Functions, Mean Value Theorem.**

10. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $\boxed{f(0) = 1}$. The function f is defined for all real numbers.

$x = 0$

$y = 1$

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



$m = 2$ $\quad$ pt $(0, 1)$ $\quad$ $y - 1 = 2(x - 1)$

(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

$y = 2x - 1$

$y(0.2) = 2(0.2) - 1$

$y = -0.6$

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

$\frac{dy}{dx} = (3-y)\cos x$

$\frac{1}{3-y}\, dy = \cos x \, dx$

$u = 3 - y$

$du = -1\, dy$

$-\ln|3-y| = \sin x + C$

$-\ln 2 = C$

$-\ln|3-y| = \sin x - \ln 2$

$e^{\ln|3-y|} = e^{\ln 2 - \sin x}$

$|3-y| = e^{\ln 2 - \sin x}$

$3 - y = e^{\ln 2 - \sin x}$

$3 - e^{\ln 2 - \sin x} = y$