

KEY

= Unit 5 Test Review #2 =

1. Express $(-2 - 3i)^3$ in simplest $a + bi$ form.

$$\begin{array}{r}
 (-2 - 3i)(-2 - 3i)(-2 - 3i) \\
 +4 +6i \\
 +6i +9i^2 \\
 \hline
 4 +12i -9 \\
 -5 +12i
 \end{array}
 \quad \rightarrow \quad
 \begin{array}{r}
 (-5 + 12i)(-2 - 3i) \\
 10 +15i \\
 -24i -30i^2 \\
 \hline
 10 -9i +36 \\
 \boxed{46 - 9i}
 \end{array}$$

2. What is the solution set of $\sqrt{x+6} - x = 4$?

$$\begin{aligned}
 & \frac{+x \quad +x}{(\sqrt{x+6})^2 = (x+4)^2} \\
 & x+6 = x^2 + 8x + 16 \\
 & 0 = x^2 + 7x + 10 \\
 & 0 = (x+5)(x+2) \\
 & x = -5 \quad x = -2
 \end{aligned}
 \quad \text{check: } \begin{aligned}
 & \sqrt{(-5)+6} - (-5) = 4 \quad \text{No!} \\
 & \sqrt{(-2)+6} - (-2) = 4 \quad \text{YES!}
 \end{aligned}
 \quad \{-2\}$$

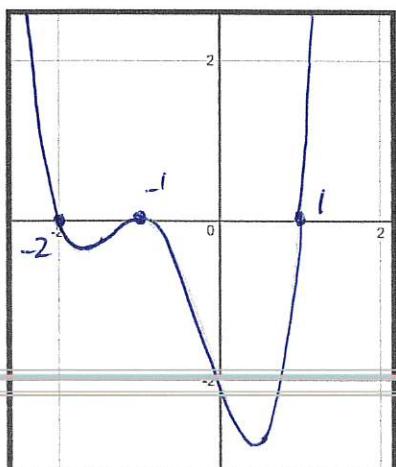
3. Determine the inverse of $f(x) = \frac{2}{3}x + 1$. Use compositions to prove that you are correct.

$$\begin{aligned}
 y &= \frac{2}{3}x + 1 \\
 x &= \frac{2}{3}y + 1 \\
 x-1 &= \frac{2}{3}y \\
 3(x-1) &= 2y
 \end{aligned}
 \quad \begin{aligned}
 \frac{3(x-1)}{2} &= y \\
 \frac{3(x-1)}{2} &= f^{-1}(x)
 \end{aligned}
 \quad \left| \begin{array}{l}
 f(f^{-1}(x)) = x \\
 f\left(\frac{3(x-1)}{2}\right) = x \\
 \frac{2}{3}\left(\frac{3(x-1)}{2}\right) + 1 = x \\
 x-1 + 1 = x \\
 x = x \checkmark
 \end{array} \right.
 \quad \left| \begin{array}{l}
 f^{-1}(f(x)) = x \\
 f^{-1}\left(\frac{2}{3}x + 1\right) = x \\
 \frac{3}{2}\left(\frac{2}{3}x + 1\right) - 1 = x \\
 \frac{2}{3}x + 1 - 1 = x \\
 \frac{2}{3}x = x \\
 x = x \checkmark
 \end{array} \right.$$

4. Solve $9x^2 + 112 = 0$ and express your answer in simplest radical form.

$$\begin{aligned}
 \frac{9x^2}{9} &= \frac{-112}{9} \\
 x^2 &= -\frac{112}{9} \\
 x &= \pm \sqrt{-\frac{112}{9}}
 \end{aligned}
 \quad \begin{aligned}
 x &= \pm i\sqrt{\frac{112}{9}} \\
 x &= \pm i\frac{\sqrt{64}\sqrt{2}}{\sqrt{9}} \\
 x &= \pm \frac{8i\sqrt{2}}{3}
 \end{aligned}$$

5. Determine the equation in standard form for the graph shown:



POSITIVE, EVEN DEGREE

ROOTS...

-2

-1

1

FACTORS...

$(x+2)$

$(x+1)^2$

$(x-1)$

SO...

$$y = (x+1)^2(x+2)(x-1)$$

$$y = (x^2 + 2x + 1)(x+2)(x-1)$$

$$\begin{aligned}
 & x^3 + 2x^2 + 1x \\
 & + 2x^2 + 4x + 2 \\
 & \hline
 & x^3 + 4x^2 + 5x + 2
 \end{aligned}$$

$$\begin{aligned}
 y &= (x^3 + 4x^2 + 5x + 2)(x-1) \\
 &= x^4 + 4x^3 + 5x^2 + 2x \\
 &\quad - x^3 - 4x^2 - 5x - 2 \\
 \hline
 y &= x^4 + 3x^3 + 1x^2 - 3x - 2
 \end{aligned}$$

TURN OVER

6. If $f(x) = 2x^2 - 10x$ and $g(x) = x - 3$, evaluate $f(g(x))$ and $(g \circ f)(x)$

$$\begin{aligned}f(g(x)) &= g(f(x)) \\f(x-3) &= 2(x-3)^2 - 10(x-3) \\&= 2(x^2 - 6x + 9) - 10(x-3) \\&= 2x^2 - 12x + 18 - 10x + 30 \\&= 2x^2 - 22x + 48 \\(g \circ f)(x) &= g(2x^2 - 10x) - 3 \\&= 2x^2 - 10x - 3\end{aligned}$$

7. Determine the inverse of $f(x) = -\frac{2}{3}x - 9$

$$\begin{aligned}y &= -\frac{2}{3}x - 9 \\x &= -\frac{2}{3}y - 9 \\x + 9 &= -\frac{2}{3}y \\3(x+9) &= -2y \\-\frac{3(x+9)}{2} &= y \\-\frac{3(x+9)}{2} &= f^{-1}(x)\end{aligned}$$

8. Solve algebraically for the exact values of x : $\frac{x}{x-1} - \frac{2}{x} = \frac{1}{x-1}$

$$\begin{aligned}\left(\frac{x}{x-1} - \frac{2}{x}\right)(x-1) &= \frac{1}{x-1}(x-1) \\x^2 - 2x - 2 &= x \\x^2 - 3x + 2 &= 0 \\(x-2)(x-1) &= 0 \\x=2, x=1, \text{ but } x \neq 1, \text{ so...} &\{ 2 \}\end{aligned}$$

9. Express in simplest form: $\sqrt[3]{-\frac{27x^{10}y^5}{81xy}}$

$$\begin{aligned}\frac{\sqrt[3]{-27} \sqrt[3]{x^{10}} \sqrt[3]{y^5}}{\sqrt[3]{81} \sqrt[3]{x} \sqrt[3]{y}} \\-\frac{3x^3 \sqrt[3]{x} y \sqrt[3]{y^2}}{3\sqrt[3]{3} \sqrt[3]{x} \sqrt[3]{y}}\end{aligned}$$

$$\begin{aligned}-\frac{x^3 y \sqrt[3]{y}}{\sqrt[3]{3}} \\-x^3 y \sqrt[3]{\frac{y}{3}}\end{aligned}$$

10. Classify the symmetry of $f(x) = -2x^9 + 4x^3 - 8x + 1$ as even, odd, or neither. Justify your answer.

EVEN: $f(x) = f(-x)$

$$\begin{aligned}-2x^9 + 4x^3 - 8x + 1 &= -2(-x)^9 + 4(-x)^3 - 8(-x) + 1 \\-2x^9 + 4x^3 - 8x + 1 &\neq 2x^9 - 4x^3 + 8x + 1\end{aligned}$$

NOT EVEN

ODD: $f(x) = -f(-x)$

$$\begin{aligned}-2x^9 + 4x^3 - 8x + 1 &= -(2x^9 - 4x^3 + 8x + 1) \\-2x^9 + 4x^3 - 8x + 1 &\neq -2x^9 + 4x^3 - 8x - 1\end{aligned}$$

NOT ODD

NEITHER

11. Determine if $x+3$ is a factor of $x^3 + 2x^2 - 5x - 6$. Justify your answer.

FACTOR $x+3$

ROOT $+3$

$$\begin{array}{r} +3 \longdiv{1 \ 2 \ -5 \ -6} \\ \downarrow +3 \quad 1.5 \quad 30 \\ 1 \ 5 \quad 10 \quad 24 \end{array}$$

SINCE THE REMAINDER IS NOT 0, $x+3$ is NOT A FACTOR OF $x^3 + 2x^2 - 5x - 6$.

TURN OVER