Polynomials & Expressions

1. The expression $\left(\frac{3}{2}x+1\right)\left(\frac{3}{2}x-1\right)-\left(\frac{3}{2}x-1\right)^2$ is equivalent to

(1) 0

(2) $-3x$

(3) $\frac{3}{4}x-2$

(4) $3x-2$

$\frac{9}{4}x^2-1 - \left(\frac{3}{2}x-1\right)\left(\frac{3}{2}x-1\right)$

$\frac{9}{4}x^2-1 - \left(\frac{9}{4}x^2-3x+1\right)$

$\frac{9}{4}x^2-1 - \frac{9}{4}x^2+3x-1$

$3x-2$

Exponents & Radicals

Express $\left(2x\sqrt{2}-y\right)^2$ as a trinomial in simplest form.

$(2x\sqrt{2}-y)(2x\sqrt{2}-y)$

$4x^2(2) - 4xy\sqrt{2}+y^2$ ⟶ $8x^2-4xy\sqrt{2}+y^2$

Simplify each so your answer has only positive (fractional) exponents.

a. $\left(3x^{\frac{1}{4}}\right)^{-2}$

$3^{-2}\cdot x^{-\frac{1}{2}} = \frac{1}{9\sqrt{x}}$

b. $\sqrt[3]{x}\cdot\sqrt[3]{-8x^2}\cdot\sqrt[3]{27x^4}$

$x^{\frac{1}{3}}\cdot\left(-2x^{\frac{2}{3}}\right)\cdot\left(3x^{\frac{4}{3}}\right)$

$-6x^{\frac{7}{3}} = -6\sqrt[3]{x^7}$

Factoring

Factor the expression x^4+6x^2-7 completely.

$(x^2+7)(x^2-1)$

$(x^2+7)(x+1)(x-1)$

Real vs. Complex

To determine the value of $(2 + i)^2$, Lily performs the following operations:

$$(2 + i)^2$$
$$= 4 + i^2$$
$$= 4 + (-1)$$
$$= 3$$

Lily made an error. Identify and explain Lily's error. Then correctly solve the problem.

$(2+i)^2 = (2+i)(2+i) = 4 + 4i + i^2$
$$= 4 + 4i - 1$$
$$= 3 + 4i$$

Lily did not expand $(2+i)^2$ correctly.

Functions

Which function has a greater average rate of change on [-2, 4]?

$g(x) = 4x^3 - 5x^2 + 3$

$g(-2) = -49$
$g(4) = 179$

$AROC = \dfrac{179 - -49}{4 - -2} = \dfrac{228}{6} = 38$
$g(x)$

x	f(x)
-2	1.25
-1	2.5
0	5
1	10
2	20
3	40
4	80

$AROC = \dfrac{80 - 1.25}{4 - -2} = 13.125$
$f(x)$

$g(x)$ has a greater AROC

Given that $f(x) = 2x + 1$, find $g(x)$ if $g(x) = 2[f(x)]^2 - 1$.

$g(x) = 2(2x+1)^2 - 1$
$$= 2(2x+1)(2x+1) - 1$$
$$= 2(4x^2 + 4x + 1) - 1$$
$$= 8x^2 + 8x + 2 - 1 = 8x^2 + 8x + 1$$

Linear & Quadratic Functions

Solve the following system of equations by elimination:

$$6x + 8y - 6z = 62$$
$$10x - 12y - 14z = 14$$
$$12x - 8y + 20z = -68$$

Get rid of y

$$3(6x + 8y - 6z = 62)$$
$$2(10x - 12y - 14z = 14)$$

$$18x + 24y - 18z = 186$$
$$20x - 24y - 28z = 28$$

$$38x - 46z = 214$$

$$6x + 8y - 6z = 62$$
$$12x - 8y + 20z = -68$$

$$18x + 14z = -6$$

$$18(38x - 46z = 214)$$
$$-38(18x + 14z = -6)$$

$$684x - 828z = 3852$$
$$-684x - 532z = 228$$

$$-1360z = 4080$$

$$\boxed{z = 3}$$

$$18x + 14(3) = -6$$
$$18x - 42 = -6$$
$$18x = 36$$
$$\boxed{x = 2}$$

$$6(2) + 8y - 6(3) = 62$$
$$12 + 8y + 18 = 62$$
$$8y + 30 = 62$$
$$8y = 32 \quad \boxed{y = 4}$$

Solve the system of equations shown below algebraically.

$$5x + 5y = 20$$
$$x^2 + y^2 - 4x = 0$$

$$5y = -5x + 20$$
$$y = -x + 4$$

$$x^2 + (-x+4)^2 - 4x = 0$$
$$x^2 + x^2 - 8x + 16 - 4x = 0$$
$$2x^2 - 12x + 16 = 0$$
$$x^2 - 6x + 8 = 0$$
$$(x-2)(x-4) = 0$$
$$x = 2 \quad x = 4$$

$$x = 2$$
$$y = -x + 4$$
$$y = -2 + 4$$
$$y = 2$$
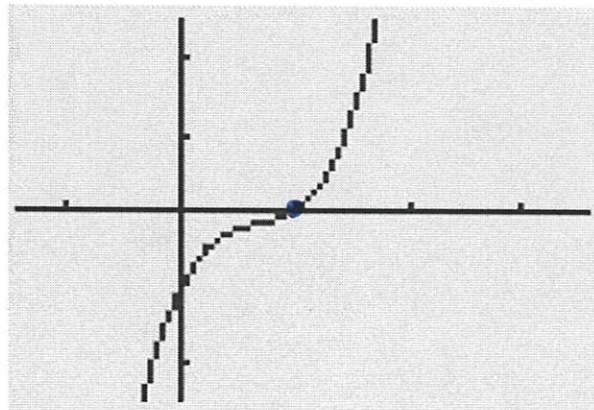$$(2, 2)$$

$$x = 4$$
$$y = -x + 4$$
$$y = -4 + 4$$
$$y = 0$$
$$(4, 0)$$

Polynomial Functions

Consider the graph of the polynomial function $P(x) = 2x^3 - 4x^2 + 3x - 1$ given below.

Use the graph to find a real zero of P.

$$x = 1$$

Confirm that the zero is correct by evaluating the function P algebraically at that value.

$$P(1) = 2(1)^3 - 4(1)^2 + 3(1) - 1$$
$$P(1) = 2 - 4 + 3 - 1 = 0$$

Using this zero, find a quadratic factor of P.

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1 | 2   -4    3   -1
  |       2   -2    1
  ----------------------
    2   -2    1    0
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$$\boxed{2x^2 - 2x + 1}$$

Find algebraically the zeros for $p(x) = x^3 + x^2 - 4x - 4$. Then, on the set of axes below, sketch the graph of $y = p(x)$.

$$x^3 + x^2 - 4x - 4 = 0$$
$$x^2(x+1) - 4(x+1) = 0$$
$$(x^2 - 4)(x+1) = 0$$
$$(x+2)(x-2)(x+1) = 0$$
$$x = -2 \quad x = 2 \quad x = -1$$