Algebra 2 Regents Review Packet #5

Key

If the point $(4, -2)$ lies on the graph of $y = f(x)$, then which of the following points must lie on the graph of its inverse, i.e. $y = f^{-1}(x)$?

Inverse functions (switch x & y)

(1) $(-2, 4)$

(2) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

(3) $(-4, 2)$

(4) $(4, 2)$

Solve the following system of equations algebraically.

$$3x - 5y + 2z = -5$$
$$\boxed{5x + y + 6z = 33}$$
$$-2x + 10y - 3z = 40$$

get rid of y

$5(5x + y + 6z = 33)$
$3x - 5y + 2z = -5$

$25x + \cancel{5y} + 30z = 165$
$3x - \cancel{5y} + 2z = -5$

$28x + 32z = 160$

$-10(5x + y + 6z = 33)$
$-2x + 10y - 3z = 40$

$-50x - \cancel{10y} - 60z = -330$
$-2x + \cancel{10y} - 3z = 40$

$-52x - 63z = -290$

$52(28x + 32z = 160)$
$28(-52x - 63z = -290)$

$\cancel{1456}x + 1664z = 8320$
$-\cancel{1456}x - 1764z = -8120$

$-100z = 200$

$\boxed{z = -2}$

$28x + 32(-2) = 160$
$28x - 64 = 160$
$28x = 224$
$\boxed{x = 8}$

$5(8) + y + 6(-2) = 33$
$40 + y - 12 = 33$
$y + 28 = 33$
$\boxed{y = 5}$

The quadratic function $f(x)$ has a turning point at $(5, -8)$. If $g(x) = f(x+7) - 3$, then at which of the following does $g(x)$ have a turning point?

left 7 ; down 3

$(-2, -11)$

(1) $(-2, -11)$

(2) $(12, -11)$

(3) $(-7, -3)$

(4) $(12, -5)$

Given the cubic polynomial $f(x) = x^3 - 5x^2 - 4x + 20$ answer the following.

(a) Find the x-intercepts of this function algebraically. Show how you arrived at your answer.

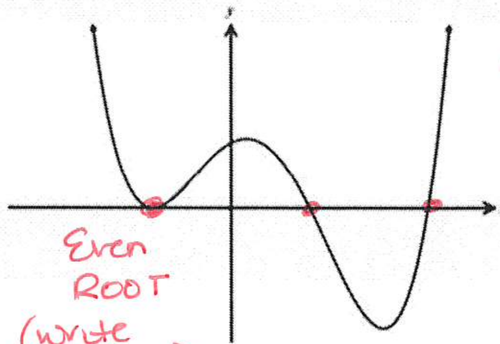
$x^2(x-5) - 4(x-5) = 0$

$(x^2-4)(x-5) = 0$

$(x+2)(x-2)(x-5) = 0$

$\boxed{x = -2 \quad x = 2 \quad x = 5}$

(b) Explain why the graph below could *not* represent that of $f(x)$.



Even ROOT (write TWICE)

This function has 4 roots, not 3, so it can't represent $f(x)$

The rational expression $\dfrac{4x^3 - 2x^2 + 8x + 10}{x-5}$ can be written as $p(x) + \dfrac{k}{x-5}$, where $p(x)$ is a quadratic polynomial and k is a constant.

(a) Determine the equation for $p(x)$. Show how you arrived at your answer.

$$\begin{array}{r|rrrr} 5 & 4 & -2 & 8 & 10 \\ & \downarrow & 20 & 90 & 490 \\ \hline & 4 & 18 & 98 & 500 \end{array}$$

$p(x) = 4x^2 + 18x + 98$

$k = 500$

(b) Is $x-5$ a factor of $4x^3 - 2x^2 + 8x + 10$? Explain how your reasoning.

No, remainder $\neq 0$ when you divide

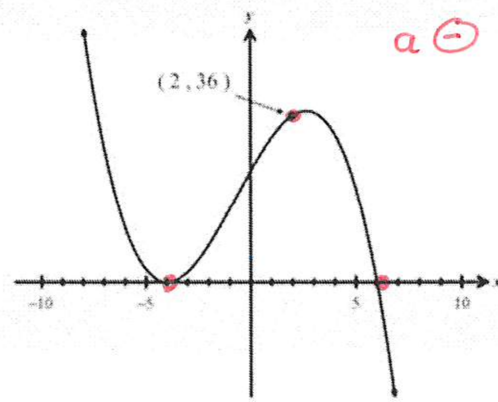
The cubic polynomial below has zeroes at $x = -4$ and $x = 6$ only and passes through the point $(2, 36)$ as shown. Algebraically determine its equation in factored form. Show how you arrived at your answer.

$x = -4 \quad x = -4 \quad x = 6$

$(x+4)(x+4)(x-6)$

$y = -\frac{1}{4}(x+4)(x+4)(x-6)$

a ⊝



$* x \neq 0, -7 *$

What is the solution set of the equation $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$?

(1) $\left\{\frac{3}{2}, 7\right\}$

(2) $\left\{\frac{7}{2}, -3\right\}$

(3) $\left\{-\frac{3}{2}, 7\right\}$

(4) $\left\{-\frac{7}{2}, -3\right\}$

LCD : $x(x+7)$

$$\frac{x(3x+25)-5(x)(x+7)}{x(x+7)} = \frac{3(x+7)}{x(x+7)}$$

$$3x^2+25x-5x^2-35x = 3x+21$$

$$-2x^2-10x = 3x+21$$

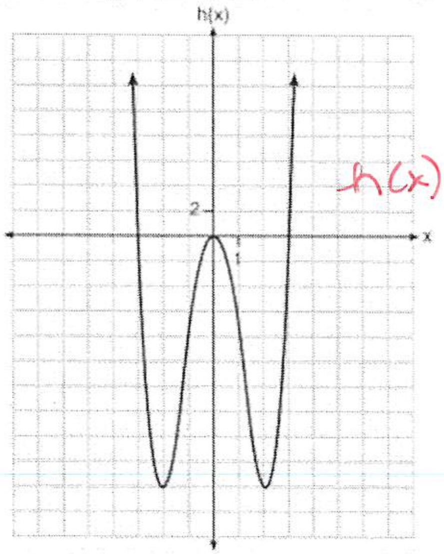
$$0 = 2x^2+13x+21$$

$$0 = (2x+7)(x+3)$$

$$x = -\frac{7}{2} \quad x = -3$$

Function f, g, and h are given below.

$$f(x) = \sin(2x)$$
$$g(x) = f(x) + 1$$



$f(-x) = \sin(-2x)$ Odd - Reflect through origin

$g(-x) = \sin(-2x)+1$ Neither

$h(x)$ even (reflect over y-axis)

Which statement is true about function f, g, and h?

(1) $f(x)$ and $g(x)$ are odd, $h(x)$ is even.

(2) $f(x)$ and $g(x)$ are even, $h(x)$ is odd.

(3) $f(x)$ is odd, $g(x)$ is neither, $h(x)$ is even.

(4) $f(x)$ is even, $g(x)$ is neither, $h(x)$ is odd.

The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are

(1) $-6 \pm 2i$

(2) $-6 \pm 2\sqrt{19}$

(3) $\boxed{6 \pm 2i}$

(4) $6 \pm 2\sqrt{19}$

$$0 = \frac{1}{2}x^2 - 6x + 20$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(\frac{1}{2})(20)}}{2(\frac{1}{2})}$$

$$x = 6 \pm \sqrt{-4}$$

$$x = 6 \pm 2i$$

Which statement is *incorrect* for the graph of the function
$y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7$?

(1) The period is 6.

(2) The amplitude is 3.

(3) The range is $[4,10]$

(4) The midline is $y = -4$.

$a = 3$ amp

$b = \pi/3$ freq — per $= \frac{2\pi}{\pi/3} = 6$

$c = -4$ HS (Right 4)

$d = 7$ VS (midline $y=7$) — max $= 10$, min $= 4$

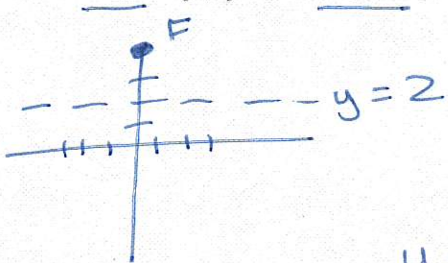
Which equation represents a parabola with a focus of $(0,4)$ and a directrix of $y = 2$?

(1) $y = x^2 + 3$

(2) $y = -x^2 + 1$

(3) $y = \frac{x^2}{2} + 3$

(4) $y = \frac{x^2}{4} + 3$



Vertex $(0,3)$

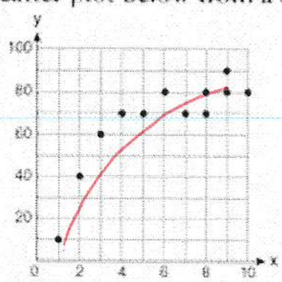
$p = 1$

a (+)

$y = \frac{1}{4(1)}(x-0)^2 + 3$

$y = \frac{1}{4}x^2 + 3$

Samantha constructs the scatter plot below from a set of data.



(diving board)

Based on her scatter plot, which regression model would be most appropriate?

(1) Exponential

(2) Linear

(3) Logarithmic

(4) Power

The expression $\frac{\sin^2 x + \cos^2 x}{1 - \sin^2 x}$ is equivalent to

(1) $\cos^2 x$

(2) $\sin^2 x$

(3) $\sec^2 x$

(4) $\csc^2 x$

$\frac{1}{\cos^2 x} = \sec^2 x$

When $\frac{3}{2}x^2 - \frac{1}{4}x - 4$ is subtracted from $\frac{5}{2}x^2 - \frac{3}{4}x + 1$, the difference is

(1) $-x^2 + \frac{1}{2}x - 5$

(2) $x^2 - \frac{1}{2}x + 5$

(3) $-x^2 - x - 3$

(4) $x^2 - x - 3$

$$- \frac{3}{2}x^2 + \frac{1}{4}x + 4$$

$$1x^2 - \frac{1}{2}x + 5$$

The expression $x^{-\frac{2}{5}}$ is equivalent to

(1) $-\sqrt{x^5}$

(2) $-\sqrt[5]{x^2}$

(3) $\frac{1}{\sqrt{x^5}}$

(4) $\frac{1}{\sqrt[5]{x^2}}$

$$\frac{1}{x^{\frac{2}{5}}} = \frac{1}{\sqrt[5]{x^2}}$$

What is the value of x in the equation $9^{3x+1} = 27^{x+2}$?

(1) 1

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) $\frac{4}{3}$

$$(3^2)^{3x+1} = (3^3)^{x+2}$$

$$2(3x+1) = 3(x+2)$$

$$6x+2 = 3x+6$$

$$3x = 4$$

$$x = \frac{4}{3}$$

Expressed in simplest form, $\frac{3y}{2y-6} + \frac{9}{6-2y}$ is equivalent to

(1) $\frac{-6y^2+36y-54}{(2y-6)(6-2y)}$

(2) $\frac{3y-9}{2y-6}$

(3) $\frac{3}{2}$

(4) $-\frac{3}{2}$

$$\frac{3y-9}{2y-6} = \frac{3(y-3)}{2(y-3)}$$

Algebraically determine the values of x that satisfy the system of equations below.

$$y = -2x + 1$$
$$y = -2x^2 + 3x + 1$$

$$-2x+1 = -2x^2+3x+1$$

$$2x^2 - 5x = 0$$

$$x(2x-5) = 0$$

$$x = 0, \frac{5}{2}$$

$x = 0$

$y = 1$

$x = \frac{5}{2}$

$y = -2(\frac{5}{2}) + 1$

$y = -4$

$\boxed{(0,1), (\frac{5}{2}, -4)}$

The results of a poll of 200 students are shown in the table below:

	Preferred Music Style		
	Techno	Rap	Country
Female	54	25	27
Male	36	40	18

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

$P(F|T) \stackrel{?}{=} P(F)$

$\frac{P(F \cap T)}{P(T)} \stackrel{?}{=} P(F)$

$\frac{54/200}{90/200} \stackrel{?}{=} \frac{106}{200}$

$\frac{54}{90} \stackrel{?}{=} \frac{106}{200}$

$.6 \neq .53$

No - not independent

Solve for x: $(\sqrt{x-4})^2 = \left(\frac{x}{4}\right)^2$

$x - 4 = \frac{x^2}{16}$

$16x - 64 = x^2$

$0 = x^2 - 16x + 64$

$0 = (x-8)(x-8)$

$x = 8$

$\{8\}$

Given: $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$

$k(x) = -|0.7x| + 5$

3 solutions (intersections)

$x = -5.17$

$x = -1.13$

$x = 1.75$

State the solutions to the equation $h(x) = k(x)$, rounded to the *nearest hundredth*.

$\boxed{2^{\text{nd}}}$ $\boxed{\text{TRACE}}$ $\boxed{5}$

Use the properties of rational exponents to determine the value of y for the equation:

$$\frac{\sqrt[3]{x^8}}{(x^4)^{\frac{1}{3}}} = x^y, \qquad x > 1$$

$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} = x^y$

$x^{\frac{4}{3}} = x^y$

$\boxed{y = \frac{4}{3}}$

33. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + (T_o - T_a)e^{-kt}$$

T_a = the temperature surrounding the object 325
T_o = the initial temperature of the object 68°
t = the time in hours
T = the temperature of the object after t hours
k = decay constant

The turkey reaches the temperature of approximately 100°F after 2 hours. Find the value of k, to the *nearest thousandth*, and write an equation to determine the temperature of the turkey after t hours.

$$100 = 325 + (68 - 325)e^{-k(2)}$$

$$-225 = -257e^{-2k}$$

$$\ln\left(\frac{225}{257}\right) = \cancel{\ln e}^{-2k})$$

$$-.1329.. = -2k$$

$$k = .06648...$$

$$\boxed{k = .066}$$

Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

$$T = 325 + (68 - 325)e^{-.066(7)}$$

$$T = 163.08...$$

$$\boxed{T = 163°}$$

34. Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year.

A summary of the two groups' final grades are shown below.

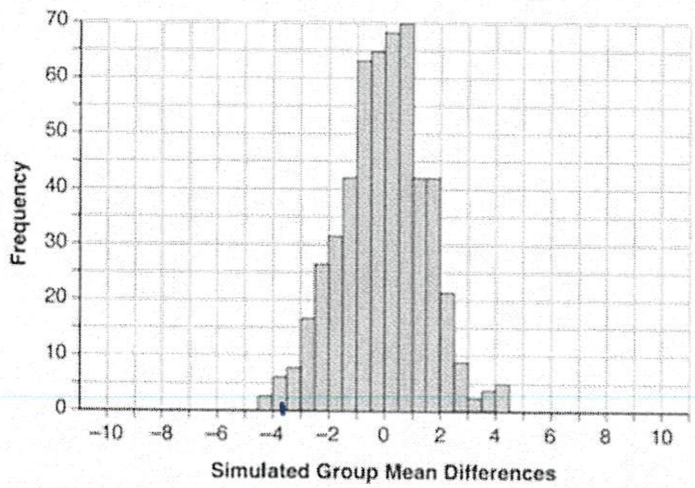
	Group 1	Group 2
$\bar{x}$	80.16	83.8
S_x	6.9	5.2

Group 1's final grade average was 3.64 pts. lower than Group 2.

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem.

$80.16 - 83.8 = -3.64$

A simulation was conducted in which the students' final grades were randomized 500 times. The results are shown below.



Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

Yes there is a significant difference because only about 2% of differences, due to chance, are less than -3.5.

35. Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$ and $\boxed{z(-5) = 0}$, algebraically determine all of the zeros of $z(x)$.

$z(-5) = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 = 0$

$-750 + 25b + 260 + 15 = 0$

$25b - 475 = 0$

$25b = 475$

$\boxed{b = 19}$

$6x^3 + 19x^2 - 52x + 15 = z(x)$

$$\begin{array}{r|rrrr} -5 & 6 & 19 & -52 & 15 \\ & \downarrow & -30 & 55 & -15 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

$(6x^2 - 11x + 3)(x+5) = z(x)$

$(2x-3)(3x-1)(x+5) = 0$

$\boxed{x = \frac{3}{2} \quad x = \frac{1}{3} \quad x = -5}$

36. Two version of a standardized test were given, an April version and a May version. The statistics for the April version shown a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed.

Joanne took the April version and score in the interval 510-540. What is the probability, to the *nearest ten thousandth*, that a test paper selected at random from the April version scored in the same interval?

normalcdf(510, 540, 480, 24) = .099440...

$\boxed{.0994}$

Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

$$z = \frac{510-480}{24} = \frac{30}{24} = \frac{5}{4} \qquad 1.25 = \frac{x-510}{20} = 535$$

$$z = \frac{540-480}{24} = \frac{60}{24} = \frac{5}{2} \qquad 2.5 = \frac{x-510}{20} = 560$$

$\boxed{535-560}$