

INTERMEDIATE VALUE THEOREM WORKSHEET

Key

Odds

In 1-4, explain why the function has a zero in the given interval.

1	$f(x) = \frac{1}{16}x^4 - x^3 + 3; [1, 2]$	$f(1) = \frac{33}{16}$ $f(2) = -4$ Since $f(x)$ is cont. on $[1, 2]$ & $f(1) = \frac{33}{16}$ & $f(2) = -4$, there is at least 1 value of x where $f(x) = 0$.
2	$f(x) = x^3 + 3x - 2; [0, 1]$	$f(0) = -1$ $f(1) = 7.728$ SAME REASON AS #1
3	$f(x) = x^2 - x - \cos x; [0, \pi]$	$f(0) = -1$ $f(\pi) = 7.728$ SAME REASON AS #1
4	$f(x) = -\frac{4}{x} + \tan\left(\frac{\pi x}{8}\right); [1, 3]$	

In 5-8, verify that the Intermediate Value Theorem guarantees that there is a zero in the interval $[0, 1]$ for the given function. Use a graphing calculator to find the zero.

5	$f(x) = x^3 + x - 1$	$f(0) = -1$ $f(1) = 1$ SAME REASON AS #1 $x = .682$
6	$f(x) = x^3 + 3x - 2$	
7	$g(t) = 2 \cos t - 3t$	$f(0) = 2$ $f(1) = -1.919$ $x = 0.504$
8	$h(\theta) = 1 + \theta - 3 \tan \theta$	

In 9-12, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem. No calculator is permitted on these problems.

9	$f(x) = x^2 + x - 1, [0, 5], f(c) = 11$	$f(0) = -1$ $f(5) = 29$ $x^2 + x - 1 = 11$ $x^2 + x - 12 = 0$ $(x+4)(x-3) = 0$ $x = -4, x = 3$ $x = 3$
10	$f(x) = x^2 - 6x + 8, [0, 3], f(c) = 0$	
11	$f(x) = x^3 - x^2 + x - 2, [0, 3], f(c) = 4$	$f(0) = -2$ $f(3) = 19$ $x^3 - x^2 + x - 2 = 4$ $x^3 - x^2 + x - 6 = 0$ $x = 2$
12	$f(x) = \frac{x^2 + x}{x - 1}, \left[\frac{5}{2}, 4\right], f(c) = 6$	

13 State how continuity is destroyed at $x = c$ for each of the graphs below:

(a) **Jump Disc.**
(no limit)

(b) **Point Disc.**
(lim exists)

(c) **Same as b.**

(d) **Same as a.**

15 Show that $f(x)$ is continuous at $x = 2$ for $f(x) = \begin{cases} 5-x, & -1 \leq x \leq 2 \\ x^2-1, & 2 < x \leq 3 \end{cases}$

$\lim_{x \rightarrow 2^-} 5-x = 3$

16 Determine whether $f(x)$ is continuous at $x = -1$ for $f(x) = \begin{cases} \frac{1}{x}, & x \leq -1 \\ x-1, & -1 < x < 1 \\ \frac{x-1}{2}, & -1 < x < 1 \\ \sqrt{x}, & x \geq 1 \end{cases}$

$\lim_{x \rightarrow -1^+} x^2 - 1 = 3$
 $\lim_{x \rightarrow -1} f(x) = 3$
 $f(-1) = 3$

∴ $f(x)$ is continuous at $x = 2$ because $\lim_{x \rightarrow 2} f(x) = f(2)$