Model

- Exponential & logs equations
- Trig equations
- Probabilities
- Statistics
- Regressions

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1. Which scenerio can be modeled by an observational study?
   a. For a class project, students in Health class ask every tenth student entering the building if they eat breakfast. SURVEY
   b. A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 districts.
   c. A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the high school, she distributes a short questionnaire in the cafeteria. SURVEY
   d. Sixty seniors taking a course in Advanced Algebra are randomly divided into two classes. One class uses the graphing calculator all the time, and the other class never uses the graphing calculator. A guidance counselor wants to determine where there is a link between graphing calculator use and students' final exam grades.

   CONTROLLED EXPERIMENT

2. The weight of a bag of potatoes at Hannaford averages 8 pounds (μ) with a standard deviation of 0.5 pound (σ). The weights of all the bags of potatoes closely follow a normal distribution. Determine what percentage of bags, to the nearest integer, weighted less than 8.25 pounds.

   2nd >VARS: 2

   high

   normalcdf(-9999, 8.25, 8, .5)

   $= .6914624678$

   69%

3. In NY, the minimum wage has grown exponentially. In 1996, the minimum wage was \$1.25 an hour and in 2015, the minimum wage was \$8.75. Algebraically determine the rate of growth to the nearest percent.

   start

   end

   $A = P(1 \pm \frac{r}{n})^{nt}$

   $\frac{8.75}{1.25} = \frac{1.25(1+r)^{19}}{1.25}$

   2015 − 1996 = 19

   $\sqrt[19]{7} = \sqrt[19]{(1+r)^{19}}$ ← 19 math: 5 ✓

   $1.1078... = 1 + r$

   $.1078... = r$

   11%

4. The speed of a tidal wave, s, in hundreds of miles per hour, can be modeled by the equation $s = \sqrt{t} - 2t + 6$, where t represents the time from its origin in hours. Algebraically determine the time when $s = 0$.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$0 = \sqrt{t} - 2t + 6$

$(2t-6)^2 = \sqrt{t}^2$

$(2t-6)(2t-6) = t$

$4t^2 - 24t + 36 = t$

$4t^2 - 25t + 36 = 0$

$a=4 \quad b=-25 \quad c=36$

$\frac{25 \pm \sqrt{49}}{8}$

$\frac{25+7}{8} = 4$ $\quad$ $\frac{25-7}{8} = 2.25$

4

5. A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug and the other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

$P(\text{drug}) = .5$

$P(\text{No drug}) = .5$

$P(\text{drug AND well}) = .4$

$P(\text{well} \mid \text{drug}) = \frac{P(\text{well AND drug})}{P(\text{drug})}$

$= \frac{.4}{.5} = .8$

6. A student created a model for the population of Detroit, $P = 714(0.75)^d$, where P is the population, in thousands, d decades after 2010. Another student, Ella, wants to use a model that would represent the population after y years. Which function below best represents Ella's model?

a. $P = 714(0.6500)^y$
b. $P = 714(0.8500)^y$
c. $P = 714(0.9716)^y$
d. $P = 714(0.9750)^y$

$\text{year} = \frac{1}{10}(\text{decade})$

$(0.75)^{\frac{1}{10}} = .97164416\ldots$

7. A formula for work problems involving two people is shown as $\frac{1}{t_1}+\frac{1}{t_2}=\frac{1}{t_b}$ where t_1 = the time taken by the first person to complete the job, t_2 = the time taken by the second person to complete the job, and t_b = the time it takes for them working together to complete the job. Fred and Barney are carpenters who build the same desk. It takes Fred 8 hours to build the desk and it takes Barney 6 hours.

   A. Write an equation that can be used to find the times it would take both carpenters working together to build the desk.

$$\frac{1}{8}+\frac{1}{6}=\frac{1}{t_b}$$

   B. Determine, to the nearest tenth of an hour, how long it would take Fred and Barney working together to build the desk.

1/8 + 1/6
math enter enter

$$\left(\frac{7}{24}\right)^{-1}=\left(\frac{1}{t_b}\right)^{-1}$$

$$\frac{24}{7}=t_b$$

3.4

8. A manufacturing company has developed a cost model, $C(x)=0.15x^3+0.01x^2+2x+120$, where x is the number of items sold, in thousands. The sales price can be modeled by $S(x)=30-0.01x$. Therefore, revenue is modeled by $R(x)=x\cdot S(x)$. Determine the company's profit, $P(x)=R(x)-C(x)$, in standard form.

$$P(x)=x(30-0.01x)-(0.15x^3+0.01x^2+2x+120)$$

$$=30x-0.01x^2-0.15x^3-0.01x^2-2x-120$$

$$=-0.15x^3-0.02x^2+28x-120$$

9. While experimenting with her calculator, Ashley creates the sequence 4, 9, 19, 39, 79, … Write a recursive sequence that models Ashley's sequence. Determine the eighth term of this sequence.

term	1	2	3	4	5
#	4	9	19	39	79
difference from prev		+5	+10	+20	+40
		+5(1)	+5(2)	+5(4)	+5(8)
		+5(2)⁰	+5(2)¹	+5(2)²	+5(2)³

$$a_1=4,\quad a_n=a_{n-1}+5(2)^{n-2}$$

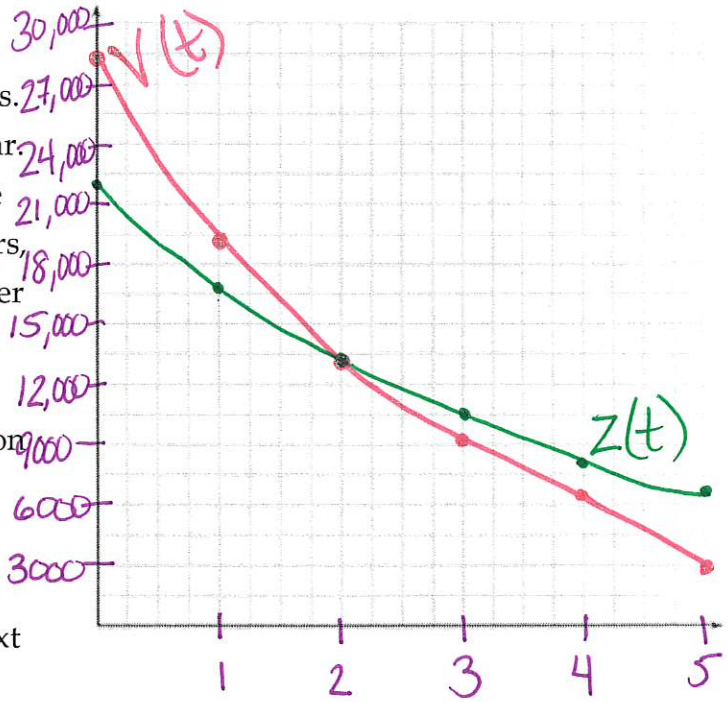
$a_6=79+5(2)^4=159$

$a_7=159+5(2)^5=319$

$a_8=319+5(2)^6=639$

← exponent is 2 less than term #

10. The value of a car based on its years in use is modeled by $V(t) = 28482.698(0.684)^t$, where V(t) is the value in dollars and t is time in years. Aidan has to take out a loan to purchase the car. The function $Z(t) = 22151.327(0.778)^t$, where Z(t) is measured in dollars and t is time in years, models the unpaid amount of Aidan's loan over time.



A. Graph V(t) and Z(t) over the interval [0, 5] on the set of axes.

B. State when V(t) = Z(t), to the nearest hundredth, and interpret its meaning in context of the problem.

2nd tableset $\Delta x = .01$

$t = 1.95$

At 1.95 years, the value of the car will be the same as how much Aidan owes on the car.

11. The function $p(t) = 110e^{0.03922t}$ models the population of a city, in millions, t years after 2010. As of today, consider the ~~following two statements~~:

I. The ~~current~~ population is 110 million — y-intercept (initial pop.)

II. The population increases continuously by approximately 3.9% per year.

This model supports:

$A = Pe^{Rt}$ $r > 0$ is inc. ✓ $r < 0$ is dec. 3.9% = .0039

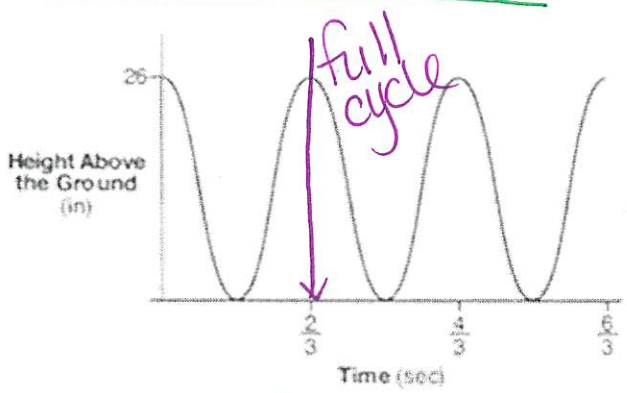
~~b.~~ I only

c. II only

~~d.~~ Both I and II

e. Neither I nor II

12. The graph below represents the height above the ground, h, in inches, of a point on a bike wheel during a ride in terms of time, t, in seconds. Identify the period of the graph and describe what the period represents in context of the story.



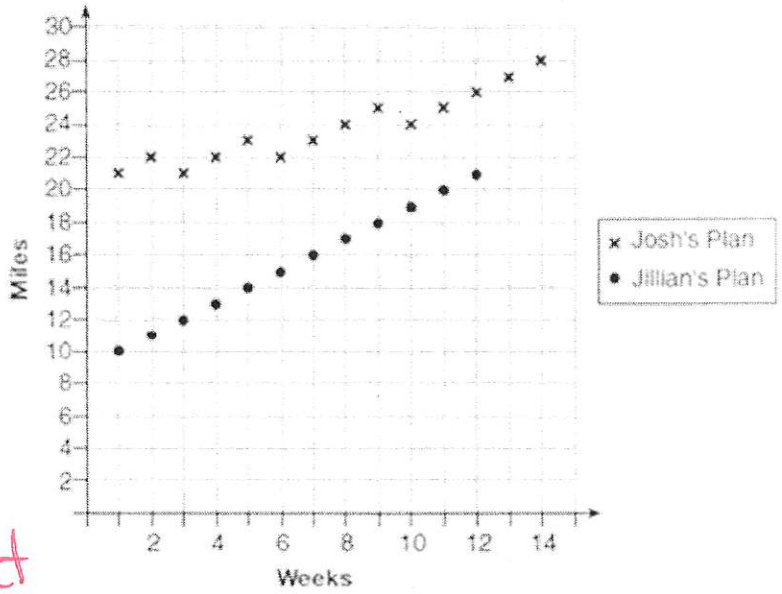
full cycle

length of one full cycle (contains max, min & starts/ends @ same height)

PER = 2/3 sec

It will take 2/3 of a second for the bike wheel to make one full revolution.

13. Olivia has decided to run a half-marathon. She is looking at 2 possible plans. Jillian's 12-week plan or Josh's 14-week plan. The number of miles run per week for each plan is shown.



A. Which one follows an arithmetic pattern? Explain your reasoning.

Jillian's b/c it has a constant rate of change.

B. Write a recursive definition to represent the number of miles run for each week for the plan you choose.

$a_1 = 10 \qquad a_n = a_{n-1} + 0.5$

19. The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts and there are 60 cycles every second. Which equation bets represents the value of voltage as it flows through the electric wires, where t is time in seconds?

a. $V = 120\sin(t)$
b. $V = 120\sin(60t)$
c. $V = 120\sin(60\pi t)$
d. $V = 120\sin(120\pi t)$

60 cycles/sec $\therefore$ $\frac{1}{60}$ sec per cycle
period

$y = A\sin(Bx) + C$ (B: Freq)

$PER = \frac{2\pi}{freq}$

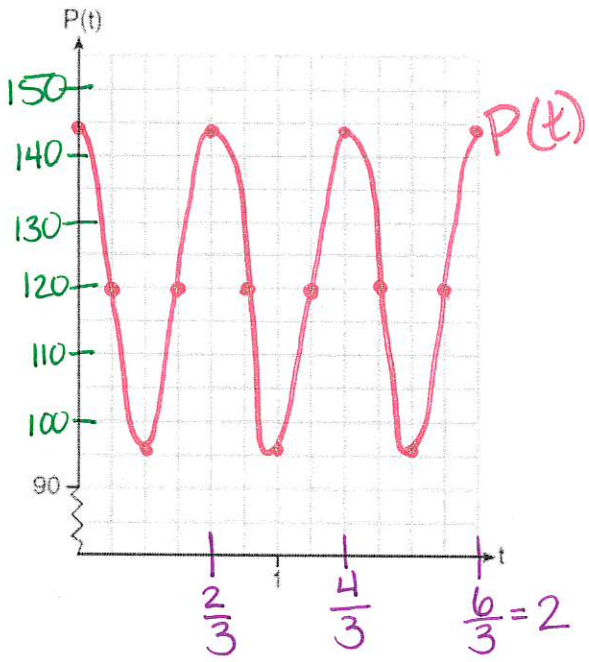
$\frac{1}{60} = \frac{2\pi}{F}$

$F = 120\pi$

20. The resting blood pressure of an adult patient can be modeled by the function $P(t) = 24\cos(3\pi t) + 120$, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds. On the set of axes, graph $y = P(t)$ over the domain [0, 2].

amp = 24
freq = 3π
PER = $\frac{2\pi}{3\pi} = 2/3$

VS(mid) = 120
max = 120 + 24 = 144
min = 120 − 24 = 96



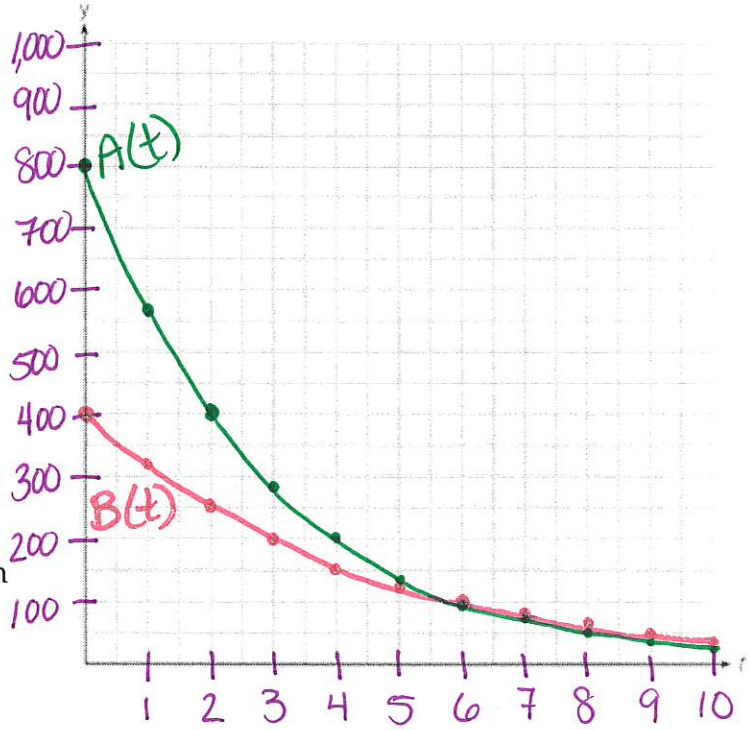
21. The breakdown of a drug is represented by $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, A(t), is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, B(t), is given 400 milligrams of the drug with a decay rate of 0.231.

A. Write two functions, A(t) and B(t), that models this situation.

$A(t) = 800e^{-.347t}$

$B(t) = 400e^{-.231t}$

B. Graph each function on the axes provided.



C. To the nearest hour, when does the amount of the drug left in Patient B begin to exceed that of Patient A?

Type A(t) into y_1
B(t) into y_2
2nd Graph

6 hours

D. The doctor will allow another 800 dosage for Patient A when 15% of the original dose is left. Determine to the nearest tenth of an hour, how long Patient A will have to wait until they receive another 800-milligram dosage.

$.15(800) = 120$

$120 = 800e^{-.347t}$

$\ln .15 = \ln e^{-.347t}$

$\ln .15 = -.347t$

$t = \frac{\ln .15}{-.347}$

5.5 hours