2018 Review Unit 4

Key

SOLVE

1. Solve algebraically for all values of x: $\sqrt{x-4}+\cancel{x}=6$

$\cancel{-x}\quad -x$

$(\sqrt{x-4})^2=(6-x)^2$

$(6-x)(6-x)$

$\cancel{x}\cancel{-4}=36-12x+x^2$

$\cancel{-x}\cancel{+4}\quad +4\quad -x$

$0=x^2-13x+40$

$0=(x-8)(x-5)$

$\cancel{x=8}\quad x=5$

CHECK!

$\{5\}$

③ 2. The value(s) of x that satisfy $\sqrt{x^2-4x-5}=2x-10$ are

1) {5}
2) {7}
$\boxed{3)\ \{5,7\}}$ ✓
~~4)~~ {3, 5, 7}

$\sqrt{5^2-4(5)-5}=2(5)-10$

$0=0$ ✓

$\sqrt{7^2-4(7)-5}=2(7)-10$

$4=4$ ✓

$\sqrt{3^2-4(3)-5}=2(3)-10$

$\sqrt{-8}\neq-4$

3. The roots of the equation $3x^2+2x=-7$ are

~~1)~~ $-2,-\frac{1}{3}$
~~2)~~ $-\frac{7}{3},1$
3) $-\frac{1}{3}\pm\frac{2i\sqrt{5}}{3}$
~~4)~~ $-\frac{1}{3}\pm\frac{\sqrt{11}}{3}$

$3x^2+2x+7=0$

$a=3\quad b=2\quad c=7$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(3)(7)}}{2(3)}$$

$$x=\frac{-2\pm\sqrt{-80}}{6}$$

eliminate choice 1, 2, 4

$\sqrt{-80}\rightarrow\sqrt{-16}\sqrt{5}$

$$x=\frac{-2\pm4i\sqrt{5}}{6}$$

$$x=\frac{-2}{6}\pm\frac{4i\sqrt{5}}{6}$$

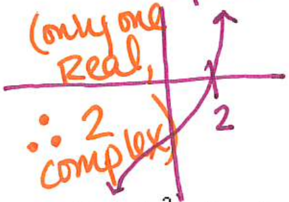
$$x=\boxed{-\frac{1}{3}\pm\frac{2i\sqrt{5}}{3}}$$

4. Given $c(m)=m^3-2m^2+4m-8$, the solution of $c(m)=0$ is

3 solutions

Graph

~~1)~~ ±2
2) 2, only
3) 2i, 2
4) ±2i, 2



(only one Real, ∴ 2 complex)

$(m^3-2m^2)+(4m-8)=0$

$m^2(m-2)+4(m-2)=0$

$(m^2+4)(m-2)=0$

$m^2+4=0\quad m-2=0$

$\sqrt{m^2}=\pm\sqrt{-4}$

$\boxed{m=\pm2i}\quad\boxed{m=2}$

5. Solve the equation $2x^2+5x+8=0$. Express the answer in $a+bi$ form.

$a=2\quad b=5\quad c=8$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-5\pm\sqrt{(5)^2-4(2)(8)}}{2(2)}$$

$i\sqrt{39}$

$$x=\frac{-5\pm\sqrt{-39}}{4}=\boxed{\frac{-5}{4}\pm\frac{\sqrt{39}}{4}i}$$

a=1 b=2 c=5

6. The roots of the equation $x^2 + 2x + 5 = 0$ are

1) -3 and 1

2) -1, only

[✓ 3) $-1 + 2i$ and $-1 - 2i$]

4) $-1 + 4i$ and $-1 - 4i$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

4i

$$x = \frac{-2}{2} \pm \frac{4i}{2}$$

$$x = -1 \pm 2i$$

SIMPLIFY

7. The expression $\frac{-3x^2 - 5x + 2}{x^3 + 2x^2}$ can be rewritten as

~~1)~~ $\frac{-3x-3}{x^2+2x}$

~~2)~~ $\frac{-3x-1}{x^2}$

~~3)~~ $-3x^{-1} + 1$

[✓ 4) $-3x^{-1} + x^{-2}$]

$$\frac{-(3x^2+5x-2)}{x^2(x+2)}$$

$$\frac{-(3x-1)\cancel{(x+2)}}{x^2\cancel{(x+2)}} \rightarrow \frac{-3x+1}{x^2} \rightarrow \frac{-3x}{x^2} + \frac{1}{x^2} \rightarrow \frac{-3}{x} + \frac{1}{x^2}$$

$$\boxed{-3x^{-1}+x^{-2}}$$

8. Which expression is equivalent to $\frac{2x^4 + 8x^3 - 25x^2 - 6x + 14}{x+6}$?

Root: -6

~~1)~~ $2x^3 + 4x^2 + x - 12 + \frac{86}{x+6}$

2) $2x^3 - 4x^2 - x + 14$

[✓ 3) $2x^3 - 4x^2 - x + \frac{14}{x+6}$]

4) $2x^3 - 4x^2 - x$

-6	2	8	-25	-6	14
	↓	-12	24	6	0
	2	-4	-1	0	⑭

9. Which expression is equivalent to $(3k - 2i)^2$, where i is the imaginary unit?

1) $9k^2 - 4$

2) $9k^2 + 4$

[✓ 3) $9k^2 - 12ki - 4$]

4) $9k^2 - 12ki + 4$

$(3k-2i)(3k-2i)$

$i^2 = -1$

$9k^2 \underbrace{-6ki - 6ki}+4i^2$

$9k^2 - 12ki + 4(-1)$

$9k^2 - 12ki - 4$

10. The expression $6xi^3(-4xi + 5)$ is equivalent to

1) $2x - 5i$

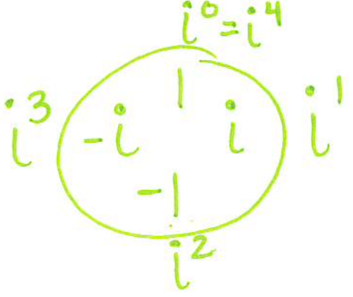
[✓ 2) $-24x^2 - 30xi$]

3) $-24x^2 + 30x - i$

4) $26x - 24x^2 i - 5i$

$-24x^2 i^4 + 30xi^3$

$-24x^2(1) + 30x(-i)$



11. Which expression is equivalent to $\frac{4x^3+9x-5}{2x-1}$, where $x \neq \frac{1}{2}$?

[✓ 1) $2x^2+x+5$]

2) $2x^2+\frac{11}{2}+\frac{1}{2(2x-1)}$

3) $2x^2-x+5$

4) $2x^2-x+4+\frac{1}{2x-1}$

$$\begin{array}{r|l} & 2x^2+x+5 \\ \hline 2x-1 & 4x^3+0x^2+9x-5 \\ & -(4x^3-2x^2) \\ \hline & 2x^2+9x \\ & -(2x^2-x) \\ \hline & 10x-5 \\ & -(10x-5) \\ \hline & 0 \end{array}$$

12. Determine the quotient and remainder when $(6a^3+11a^2-4a-9)$ is divided by $(3a-2)$. Express your answer in the form $q(a)+\frac{r(a)}{d(a)}$.

$$\begin{array}{r|l} & 2a^2+5a+2 \\ \hline 3a-2 & 6a^3+11a^2-4a-9 \\ & -(6a^3-4a^2) \\ \hline & 15a^2-4a \\ & -(15a^2-10a) \\ \hline & 6a-9 \\ & -(6a-4) \\ \hline & -5 \end{array}$$

$$\boxed{2a^2+5a+2-\frac{5}{3a-2}}$$

13. What is the quotient when $10x^3-3x^2-7x+3$ is divided by $2x-1$?

1) $5x^2+x+3$

2) $5x^2-x+3$

3) $5x^2-x-3$

[✓ 4) $5x^2+x-3$]

$$\begin{array}{r|l} & 5x^2+x-3 \\ \hline 2x-1 & 10x^3-3x^2-7x+3 \\ & -(10x^3-5x^2) \\ \hline & 2x^2-7x \\ & -(2x^2-x) \\ \hline & -6x+3 \\ & -(-6x+3) \\ \hline & 0 \end{array}$$

14. Written in simplest form, $\frac{c^2-d^2}{d^2+cd-2c^2}$ where $c \neq d$, is equivalent to

1) $\frac{c+d}{d+2c}$

2) $\frac{c-d}{d+2c}$

[✓ 3) $\frac{-c-d}{d+2c}$]

4) $\frac{-c+d}{d+2c}$

$$\frac{(c+d)\cancel{(c-d)}}{(d+2c)\cancel{(d-c)}}\ (-1)$$

$$\frac{-(c+d)}{d+2c}$$

15. If $p(x)=2x^3-3x+5$, what is the remainder of $p(x) \div (x-5)$?

$2x^3+0x^2-3x+5$

Root: 5

1) −230

2) 0

3) 40

[✓ 4) 240]

$$\begin{array}{r|rrr|r} 5 & 2 & 0 & -3 & 5 \\ & \downarrow & 10 & 50 & 235 \\ \hline & 2 & 10 & 47 & \boxed{240} \end{array}$$

16. Completely factor the following expression: $x^2 + 3xy + 3x^3 + y$ ← Re-write in standard form

$(3x^3 + x^2) + (3xy + y)$

$x^2(\underline{3x+1}) + y(\underline{3x+1})$

$\boxed{(3x+1)(x^2+y)}$

17. Given: $f(x) = \boxed{2x^2 + x - 3}$ and $g(x) = \boxed{x - 1}$

Express $f(x) \bullet g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

$[(2x^2+x-3)(x-1)] - [(2x^2+x-3) + (x-1)]$

$[2x^3 - 2x^2 + x^2 - x - 3x + 3] - [2x^2 + 2x - 4]$

$2x^3 - x^2 - 4x + 3 - 2x^2 - 2x + 4 \rightarrow \boxed{2x^3 - 3x^2 - 6x + 7}$

18. If $A = -3 + 5i$, $B = 4 - 2i$, and $C = 1 + 6i$, where i is the imaginary unit, then $A - BC$ equals

1) $5 - 17i$

2) $5 + 27i$

3) $\boxed{-19 - 17i}$

4) $-19 + 27i$

$\boxed{i^2 = -1}$

$(-3+5i) - [(4-2i)(1+6i)]$

$-3+5i - [4+24i-2i-12i^2]$

$-3+5i - [4+22i-12(-1)]$ +12

$-3+5i - [16+22i]$

$-3+5i-16-22i$

$-19-17i$

19. Over the set of integers, factor the expression $x^4 - 4x^2 - 12$.

$(x^2-6)(x^2+2)$

20. For all values of x for which the expression is defined, $\frac{x^3 + 2x^2 - 9x - 18}{x^3 - x^2 - 6x}$, in simplest form, is equivalent to

1) 3

2) $-\frac{17}{2}$

3) $\boxed{\frac{x+3}{x}}$

4) $\frac{x^2-9}{x(x-3)}$

numerator:

$(x^3+2x^2)+(-9x-18)$

$x^2(x+2)-9(x+2)$

$(x^2-9)(x+2)$

$(x+3)(x-3)(x+2)$

denominator:

$x(x^2-x-6)$

$x(x-3)(x+2)$

$$\frac{(x+3)\cancel{(x-3)}\cancel{(x+2)}}{x\cancel{(x-3)}\cancel{(x+2)}}$$

21. Where i is the imaginary unit, the expression $(x + 3i)^2 - (2x - 3i)^2$ is equivalent to

1) $-3x^2$

2) $-3x^2 - 18$

3) $\boxed{-3x^2 + 18xi}$

4) $-3x^2 - 6xi - 18$

$[(x+3i)(x+3i)] - [(2x-3i)(2x-3i)]$

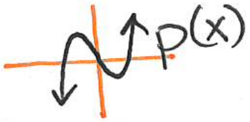
$[x^2+6xi+9\overset{=-1}{i^2}] - [4x^2-12xi+9\overset{=-1}{i^2}]$

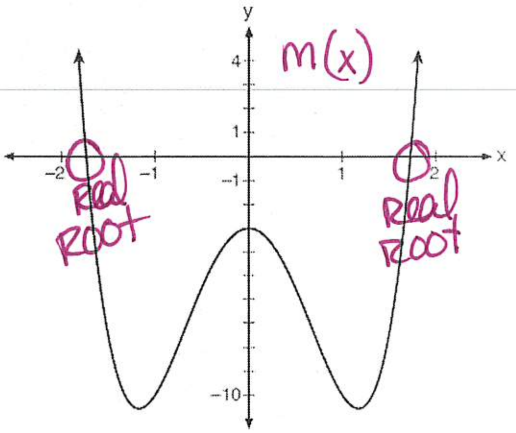
$[x^2+6xi-9] - [4x^2-12xi-9]$

$x^2+\underline{6xi}-\cancel{9}-4x^2+\underline{12xi}+\cancel{9}$

GRAPH

22. Consider the function $p(x) = 3x^3 + x^2 - 5x$ and the graph of $y = m(x)$ shown.





Which statement is true?

[✓ 1) $p(x)$ has three real roots and $m(x)$ has two real roots.]

2) $p(x)$ has one real root and $m(x)$ has two real roots.

3) $p(x)$ has two real roots and $m(x)$ has three real roots.

4) $p(x)$ has three real roots and $m(x)$ has four real roots.

23. The parabola described by the equation $y = \frac{1}{12}(x-2)^2 + 2$ has the directrix at $y = -1$. The focus of the parabola is

1) $(2,-1)$

2) $(2,2)$

3) $(2,3)$

[✓ 4) $(2,5)$]

vertex: (2, 2) ↑+3

directrix $y = -1$

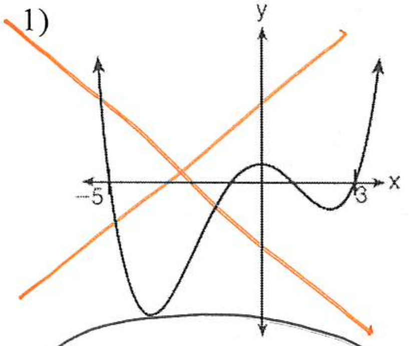
focus $(2, (2+3))$

ends same direction

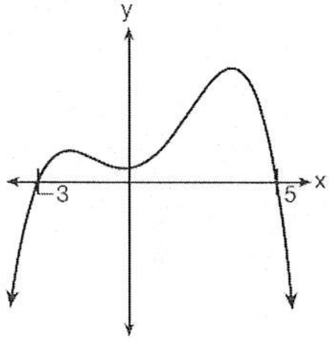
24. A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?

2 Real Roots

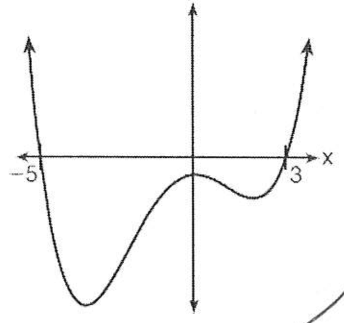
1)



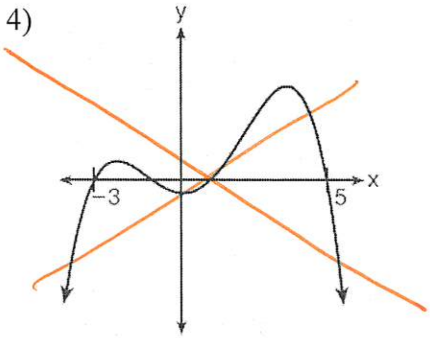
3)



[✓ 2)]

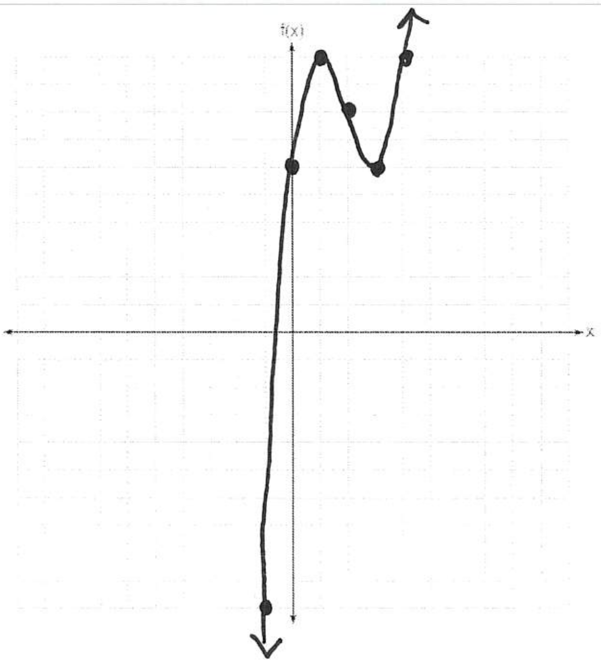


4)

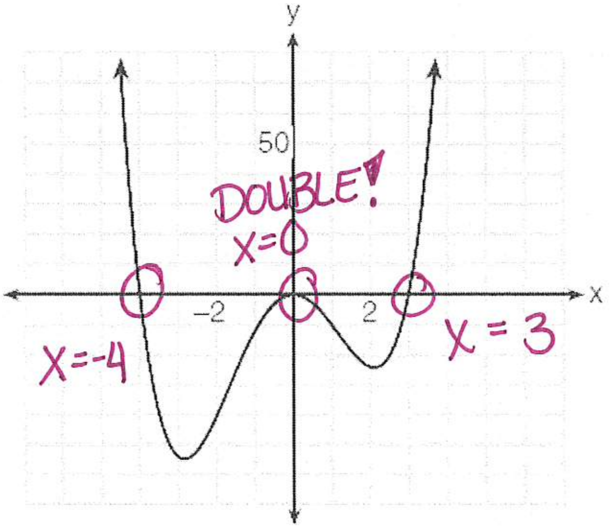


25. On the grid below, graph the function $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \le x \le 4$.

X	y
-1	-10
0	6
1	10
2	8
3	6
4	10



26. The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



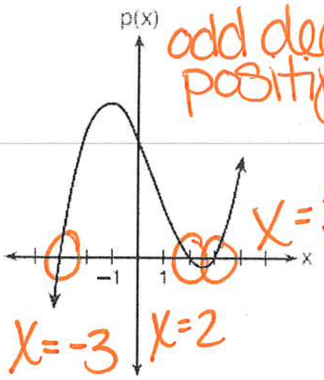
Write an equation for $f(x)$.

$f(x) = x^2(x+4)(x-3)$

27. The graph of the function $p(x)$ is sketched below.

$p(x) = \underbrace{(x+3)(x-3)}_{(x^2-9)}(x-2)$

$(x^2-9)(x-2)$



odd degree
positive

Which equation could represent $p(x)$?

[✓ 1) $p(x) = (x^2 - 9)(x - 2)$]

3) $p(x) = (x^2 + 9)(x - 2)$

2) $p(x) = x^3 - 2x^2 + 9x + 18$

4) $p(x) = x^3 + 2x^2 - 9x - 18$

EVALUATE

28. Evaluate $j(-1)$ given $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$. Explain what your answer tells you about $x + 1$ as a factor. Algebraically find the remaining zeros of $j(x)$.

$j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48$

$j(-1) = 0$ ∴ $x = -1$ is a root, which means $(x+1)$ is a factor.

ZeROS: $\{-1, \pm 4, 3/2\}$

$$\begin{array}{r|rrrr|r} -1 & 2 & -1 & -35 & 16 & 48 \\ & \downarrow & -2 & 3 & 32 & -48 \\ \hline & 2 & -3 & -32 & 48 & \textcircled{0} \end{array}$$

$(2x^3 - 3x^2) + (-32x + 48) = 0$

$x^2(2x-3) - 16(2x-3) = 0$

$(x^2-16)(2x-3) = 0$

$(x+4)(x-4)(2x-3) = 0$

$x = \pm 4, 3/2$

29. Which binomial $\boxed{\text{is } not}$ a factor of the expression $x^3 - 11x^2 + 16x + 84$?

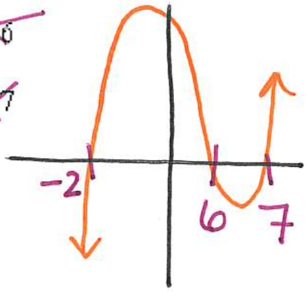
1) ~~$x + 2$~~

3) ~~$x - 6$~~

[✓ 2) $x + 4$]

4) ~~$x - 7$~~

no ROOT @ -4



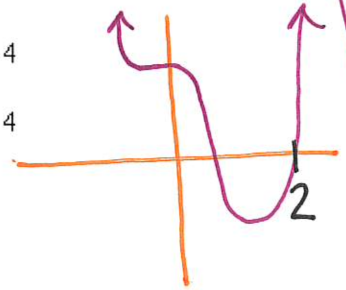
30. Which binomial is a factor of $x^4 - 4x^2 - 4x + 8$?

[✓ 1) $x - 2$]

3) $x - 4$

2) $x + 2$

4) $x + 4$



31. Given $r(x) = x^3 - 4x^2 + 4x - 6$, find the value of $r(2)$. What does your answer tell you about $x - 2$ as a factor of $r(x)$? Explain.

$r(2) = (2)^3 - 4(2)^2 + 4(2) - 6$

$r(2) = -6$

∴ $(2, -6)$ is a point on the graph, but $x = 2$ is NOT a Root, so $(x-2)$ is NOT a factor