UNIT 5 B REVIEW SHEET

Key

1. A particle moves along the x- axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the **velocity** of the particle **zero**?

   A. 1
   B. 2
   C. 3
   D. 4
   E. 5

   $v = 2t - 6$

   $v = 0$ when $t = 3$

2. The position of a particle is given by the formula, $x = -2t^3 + t^2 + 1$. At t= 1, which of the following statements is correct? Circle each CORRECT STATEMENT.

   $v = -6t^2 + 2t$ $\quad a = -12t + 2$

   I) Its speed is increasing II) Its speed is decreasing III) Acceleration = 0

   $v(1) = -4$ $\quad a(1) = -10$

3. **A particle moves along the x- axis with velocity at $t \geq 0$ given by $v(t) = -1 + e^{1-t}$. You may use your calculator!**

   a. **Find the acceleration of the particle at time t= 3.**

   $a(t) = -e^{1-x}$ $\quad a(3) = -e^{-2} = -0.135$

   b. **Is the speed of the particle increasing at time t= 3? Explain (in words) your answer.** $v(3) = -1.135$

   Speed is increasing since $v(3)$ & $a(3) > 0$

   c. **Find all values of t at which the particle changes direction. Justify your answer.**

   $v = 0$ when $t = 1$

   v: + (0) − (1)

   at $t = 1$, $v(t)$ changes sign because $v(t)$ changes sign

   d. **What is the acceleration the first time velocity = 0?**

   $a(1) = -1$

   e. Find the average acceleration on [1, 5].

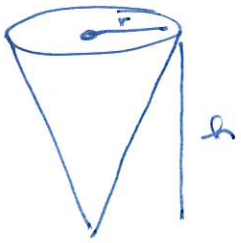
   $v(1) = 0$

   $v(5) = -.982$

   $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-.982 - 0}{5 - 1} = -.245$

when $r=2$, $V=8\pi$, $\frac{dr}{dt}=\frac{1}{3}$

4. The volume of a cone ($V=\frac{1}{3}\pi r^2 h$) is increasing at the rate of 4π cubic inches per second. At the instant when the radius of the cone is 2 inches, its volume is 8π cubic inches and the radius is increasing at 1/3 inch per second. $\frac{dV}{dt}=4\pi \text{ in}^3/\text{sec}$



a. At the instant when the radius of the cone is 2 inches, what is the rate of change of the area of its base? Include units.

$A=\pi r^2$

$\frac{dA}{dt}=2\pi r\cdot\frac{dr}{dt}$

$\frac{dA}{dt}=2\pi(2)(\frac{1}{3})$

$\frac{dA}{dt}=\frac{4}{3}\pi \text{ in}^2/\text{sec}$

$V=\frac{1}{3}\pi r^2 h$

$8\pi=\frac{1}{3}\pi(2)^2 h$

$24=4h$

$6=h$

$\frac{r}{h}=\frac{2}{6}=\frac{1}{3}$

$r=\frac{1}{3}h$

b. At the instant when the radius of the cone is 2 inches, what is the rate of change of its height? Include units.

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(\frac{1}{3}h)^2 h$

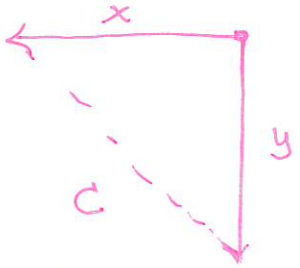
$V=\frac{\pi}{27}h^3$

$\frac{dV}{dt}=\frac{\pi}{9}h^2\cdot\frac{dh}{dt}$

$4\pi=\frac{\pi}{9}(6)^2\cdot\frac{dh}{dt}$

$1 \text{ in/sec}=\frac{dh}{dt}$

5. Mr. Sawyer is supposed to meet his wife and kids at the port of Albany for a nice boat ride one sunny afternoon. Unfortunately, Sawyer falls asleep. Sawyer's wife is so mad; she gets on the Dutch Apple Cruise Ship without him at 2:00 pm. The Dutch Apple sails west at a rate of 8 miles per hour. Sawyer gets to the port as soon as possible and accidentally gets on the JP Morgan Cruise Ship at 3:00 pm. The JP Morgan sails south at a rate of 9 miles per hour. At what rate are the ships (and Sawyer and his family) moving apart at 5:00 pm?



$\frac{dx}{dt}=8 \text{ mph}$

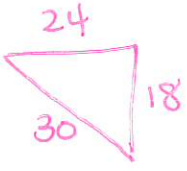
$\frac{dy}{dt}=9 \text{ mph}$

$x^2+y^2=c^2$

$2x\cdot\frac{dx}{dt}+2y\cdot\frac{dy}{dt}=2c\frac{dc}{dt}$

$24(8)+18(9)=30\cdot\frac{dc}{dt}$

$\frac{354}{30}=11.8 \text{ mph}=\frac{dc}{dt}$



at 5:00 pm

Other topics on this exam: *Limits, IVT, MVT, POI, Derivative Rules, Tangent lines, Implicit differentiation, 1st and 2nd Derivative Tests.*