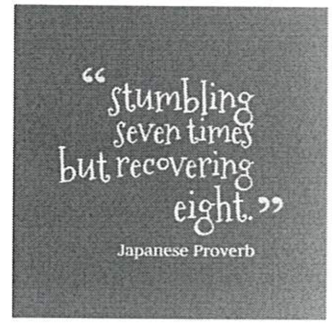
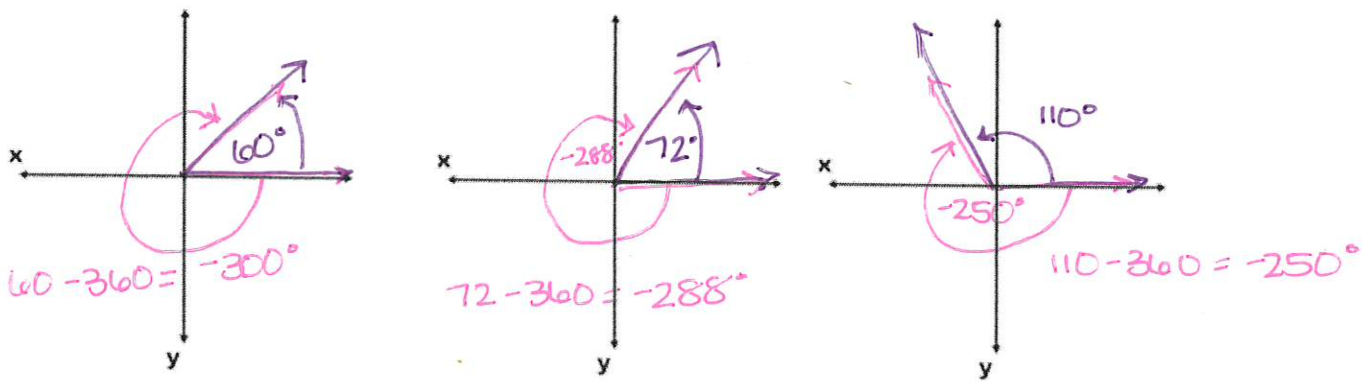
Key

Unit 6 – Trigonometry & The Unit Circle Review Sheet



1. Complete the following for each part below:
   i. Convert the following radian (if needed) measures to degrees
   ii. Draw the angles in standard position on the axes provided
   iii. State and draw a coterminal angle for each

a. $\frac{\pi}{3} = \frac{180}{3} = 60°$ b. $\frac{2\pi}{5} = \frac{2(180)}{5} = 72°$ c. $110°$



2. Indicate whether each of the functions is odd, even, or neither. Justify your answer.

   a. $f(x) = -2x^2 + 3$

   $f(-x) = -2(-x)^2 + 3 = -2x^2 + 3$ EVEN

   b. $g(x) = f(x) - 4$

   $g(-x) = -2(-x)^2 + 3 - 4 = -2x^2 - 1$ EVEN

   c. $h(x) = f(x) + 2x$

   $h(-x) = -2(-x)^2 + 3 + 2(-x)$

   $= -2x^2 - 2x + 3$ NEITHER

3. Solve the equation $\sqrt{2x-7} + x = 5$ algebraically, and justify the solution set.

$(\sqrt{2x-7})^2 = (5-x)^2$

$2x - 7 = 25 - 10x + x^2$

$0 = x^2 - 12x + 32$

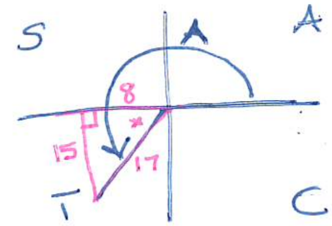
$0 = (x-4)(x-8)$

$x = 4, 8$

$\boxed{\{4\}}$

cos (−) tan (+) Quad III

4. If $\cos A = -\frac{8}{17}$ and $\tan A > 0$, find:



a. sin A

$-\frac{15}{17}$

b. tan A

$+\frac{15}{8}$

c. A to the nearest degree

$x = \tan^{-1}(15/8)$

$x = 61.92\ldots$

$A = 180 + x = 241.92\ldots$ $\boxed{242}$

$8^2 + b^2 = 17^2$

$64 + b^2 = 289$

$b^2 = 225$

$\boxed{b = 15}$

d. csc A $= \frac{1}{\sin A}$

$-\frac{17}{15}$

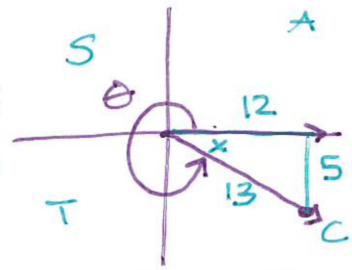
e. cot A $= \frac{1}{\tan A}$

$+\frac{8}{15}$

f. sec A $= \frac{1}{\cos A}$

$-\frac{17}{8}$

5. If the terminal side of θ, in standard position, passes through the point $(12, -5)$, what is the numerical value of:



a. $\sin\theta$

$\sin\theta = -\sin x$

$-\frac{5}{13}$

b. $\cos\theta$

$\cos\theta = \cos x$

$\frac{12}{13}$

c. $\cot\theta$

$\cot\theta = -\cot x$

$\tan\theta = -\tan x$

$= -\frac{5}{12}$

$\boxed{\cot\theta = \frac{-12}{5}}$

$12^2 + 5^2 = c^2$

$144 + 25 = c^2$

$169 = c^2$

$\boxed{13 = c}$

6. Simplify the following completely:

a. $\frac{1}{\cos^2\theta} - \tan^2\theta = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta} = 1$

b. $\sec^2 x - \sec^2 x \cdot \sin^2 x$

$\sec^2 x(1 - \sin^2 x)$

$\sec^2 x(\cos^2 x) = \frac{1}{\cos^2 x}(\cos^2 x) = 1$

7. Find the exact value of $\tan\frac{\pi}{4} - \cos\frac{\pi}{3} + \sin\frac{\pi}{3}$.

$\tan 45° - \cos 60° + \sin 60°$

$1 - \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

8. Express, in terms of i, the roots of the following function $y = \frac{1}{4}x^2 + 3$

$\frac{1}{4}x^2 + 3 = 0$

$a = \frac{1}{4}$

$b = 0$

$c = 3$

$$x = \frac{0 \pm \sqrt{0^2 - 4(\frac{1}{4})(3)}}{2(\frac{1}{4})} = \frac{0 \pm \sqrt{-3}}{\frac{1}{2}} = \boxed{0 \pm 2i\sqrt{3}}$$

9. Solve for all values of x: $\frac{x}{x-2} - \frac{9}{x} = \frac{4}{x^2-2x}$

$x \neq 0, 2$

LCD: $x(x-2)$

$$\frac{x(x) - 9(x-2)}{x(x-2)} = \frac{4}{x(x-2)}$$

$x^2 - 9x + 18 = 4$

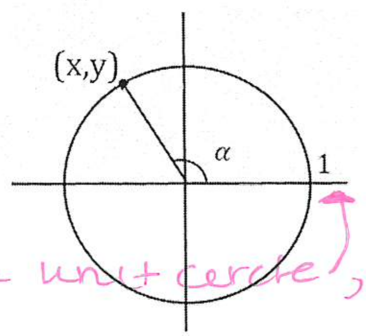
$x^2 - 9x + 14 = 0$

$(x-2)(x-7) = 0$

$x = 2, 7$

$\boxed{\{7\}}$

10. Given the graph below, justify why $sec\alpha = \frac{1}{x}$.

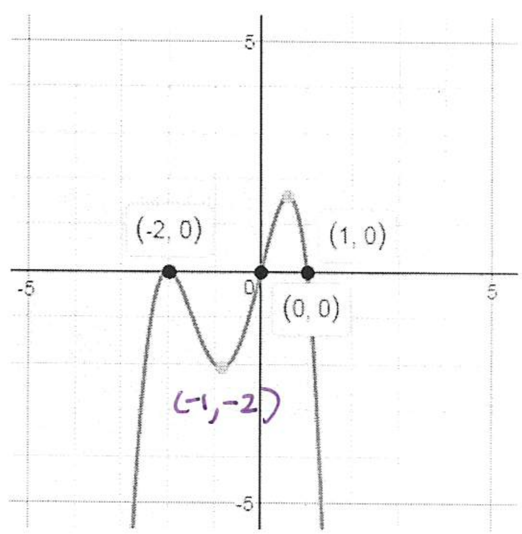


$\sec \alpha = \frac{1}{x}$

$\cos \alpha = x$

Since (x,y) is on the unit circle, $(x,y) = (\cos\alpha, \sin\alpha)$

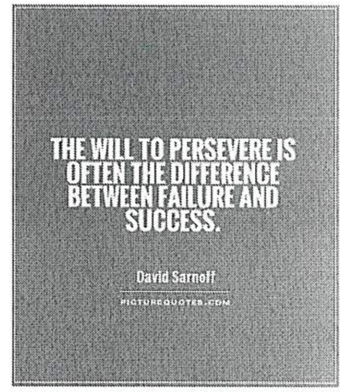
11. Given the graph of the function shown, determine the average rate of change on the interval [-2, -1].



$$AROC = \frac{\Delta y}{\Delta x} = \frac{0 - -2}{-2 - -1} = \frac{2}{-1} = -2$$

12. Solve the following system of equations algebraically.

$$(x-2)^2 + (y-1)^2 = 25$$
$$2x + y = 15$$



$y = \boxed{15-2x}$

$(x-2)^2 + (15-2x-1)^2 = 25$

$(x-2)^2 + (-2x+14)^2 = 25$

$x^2 - 4x + 4 + 4x^2 - 56x + 196 = 25$

$5x^2 - 60x + 200 = 25$

$\dfrac{5x^2 - 60x + 175}{5} = \dfrac{0}{5}$

$x^2 - 12x + 35 = 0$

$(x-5)(x-7) = 0$

$x = 5 \qquad x = 7$

$x = 5$
$y = 15 - 2(5)$
$y = 15 - 10$
$y = 5$
$\boxed{(5,5)}$

$x = 7$
$y = 15 - 2(7)$
$y = 15 - 14$
$y = 1$
$\boxed{(7,1)}$

13. Show algebraically that $x = 6$ is a root of the function $f(x) = x^3 - 3x^2 - 20x + 12$.

6	1	−3	−20	12
	↓	6	18	−12
	1	3	−2	0

OR

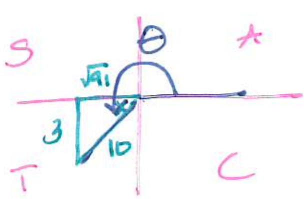
$f(6) = 6^3 - 3(6)^2 - 20(6) + 12$

$f(6) = 0$

14. Given that $sin\theta = -0.3$, and θ is an angle in quadrant III, determine the value of $cot\theta$.

$\sin\theta = \dfrac{-3}{10}$

$\tan\theta = \tan x = \dfrac{3}{\sqrt{91}}$



$3^2 + b^2 = 10^2$
$9 + b^2 = 100$
$b^2 = 91$
$b = \sqrt{91}$

$\boxed{\cot\theta = \dfrac{\sqrt{91}}{3}}$

15. Given that i is the imaginary unit, simplify the following expression completely $3i^3(2xi-4)^2$

$3i^3(2xi-4)^2 = \boxed{3i^3}\boxed{(2xi-4)(2xi-4)}$

$-3i(-4x^2 - 16xi + 16) = 12x^2i + 48x - 48i$