

Key

Review for "Derivatives..The Beginning" Exam

1. Using the table below, find the following derivatives:

x	f(x)	g(x)	f'(x)	g'(x)
-3	4	-6	-1	2
4	-1	-3	4	11

a. $\frac{d}{dx}[f(x) + g(x)]$ at $x = -3$

$$f'(x) + g'(x) = f'(-3) + g'(-3) = -1 + 2 = 1$$

b. $\frac{d}{dx}[f(x) \cdot 3g(x)]$ at $x = -3$

$$f(x)(3g'(x)) + 3g(x)(f'(x)) = 4(3(2)) + 3(-6)(-1) = 24 + 18 = 42$$

c. $\frac{d}{dx}[f(g(x))]$ at $x = 4$

$$f'(g(x)) \cdot g'(x) = f'(-3)(11) = -1(11) = -11$$

d. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ at $x = 4$

$$\frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2} = \frac{-3(4) - (-1)(11)}{(-3)^2} = \frac{-12 + 11}{9} = \frac{-1}{9}$$

2. Given the following values of a continuous function, $f(x)$ in the table below. Find the average rate of change of each given the specific interval:

x	-3	2	4	9
f(x)	7	-11	5	-1

a. $[-3, 2]$

$$AROC = \frac{-11 - 7}{2 - (-3)} = \frac{-18}{5}$$

b. $[2, 9]$

$$AROC = \frac{-1 - (-11)}{9 - 2} = \frac{10}{7}$$

c. $[-3, 9]$

$$AROC = \frac{-1 - 7}{9 - (-3)} = \frac{-8}{12}$$

3. Write the equation of the tangent line to $y = \tan x - 3x + 8$ at $x = 0$.

$P = (0, 8)$

$m = -2$

$$y' = \sec^2 x - 3$$

$$y'(0) = 1 - 3 = -2$$

$$y - 8 = -2(x - 0)$$

f. $y = \frac{3-x}{3x-1}$ $y' = \frac{(3x-1)(-1) - (3-x)(3)}{(3x-1)^2}$

g. $y = e^{\sqrt{3x}}$ $y' = e^{\sqrt{3x}} \left(\frac{1}{2}(3x)^{-\frac{1}{2}} (3) \right)$

h. $y = \cos^4(5x)$ $y' = 4(\cos^3(5x))(-\sin(5x))(5)$

i. $y = -4x^2(\sin x)$ $y' = -4x^2(\cos x) + \sin x(-8x)$

j. $y = \tan(e^{x^2})$ $y' = \sec^2(e^{x^2})(e^{x^2})(2x)$

k. $y = \left(\frac{4x+9}{3x-1} \right)^3$ $y' = 3 \left(\frac{4x+9}{3x-1} \right)^2 \left[\frac{(3x-1)(4) - (4x+9)(3)}{(3x-1)^2} \right]$

l. $y = \frac{4x^6}{3} = \frac{4}{3}x^6$ $y' = 8x^5$

8. Find all points of the function $y = x(x^2 - 6x)$ where the slope = 0. $x^3 - 6x^2$

$$y' = x(2x-6) + (x^2-6x) = 0$$

$$2x^2 - 6x + x^2 - 6x = 0$$

$$3x^2 - 12x = 0$$

$$x^2 - 4x = 0 \Rightarrow x(x-4) = 0$$

$$\begin{matrix} x=0 & x=4 \\ (0,0) & (4,40) \end{matrix}$$

9. What is the instantaneous rate of change for the function $y = \frac{3x}{x^2 + 1}$ at $x = 1$?

A) -1

B) 0

C) 1

D) -2

E) None of these

$$y' = \frac{(x^2 + 1)(3) - 3x(2x)}{(x^2 + 1)^2}$$

$$y'(1) = \frac{2(3) - 3(2)}{4}$$