Key

UNIT 4 REVIEW SHEET #1

1. Determine the **zeros (roots)** of the function $y = x^3 + 2x^2 - 5x - 6$. 3 roots

   graph

   A. -3, -1, 2
   B. -3, -1, 4
   C. -1, 2, 3
   D. -3, 2, 0

   $x = -3, -1, 2$

   (all real roots)

2. Which polynomial equation has **roots** at -1, 2i, and -2i? * Guess & check or write equation!

   A. $x^3 + x^2 + 4x + 4 = 0$
   B. $3x + x^2 + 4x - 4 = 0$
   C. $x^3 + x^2 - 4x + 4 = 0$
   D. $x^3 - x^2 + 4x + 4 = 0$

   $x = -1 \quad x = 2i \quad x = -2i$

   $(x+1)\boxed{(x-2i)(x+2i)}$

   $(x+1)(x^2+4) = x^3 + 4x + x^2 + 4$

3. A. Determine the **quotient and the remainder** when you divide $(x^4 - 3x^2 + 2x - 1)$ by $(x - 1)$.

   Syn. or Long

1	1	0	-3	2	-1
	↓	1	1	-2	0
	1	1	-2	0	-1

   Remainder

   $\underbrace{x^3 + x^2 - 2x}_{\text{Quotient}} - \frac{1}{x-1}$

   B. Is $(x - 1)$ a **factor** of $(x^4 - 3x^2 + 2x - 1)$? **Explain your reasoning.**

   No because when I divided $x^4 - 3x^2 + 2x - 1$ by $x - 1$, there was a remainder.

4. Determine the number of **real roots** and the number of **complex roots** for each function. **Explain your answer for each:** Cross x-axis

   A. $3x^5 + 2x^3 + x^2 + 7x - 1 = 0$ ← Graph these → B. $f(x) = x^5 - 4x^2 + 2x - 1$

   5 roots → 1 real, 4 complex

   5 roots → 1 Real, 4 complex

5. Given the function, $f(x) = x^3 + x^2 + x - 3$.

A. **How many roots** *should* this function have? How many are **real**? How many are **complex**?

3 roots $\to$ 1 Real, 2 complex

B. State the **real** root.

$x = 1$

C. Write the function as a **product of a linear factor and a quadratic factor**.

$$\begin{array}{r|rrrr} 1 & 1 & 1 & 1 & -3 \\ & \downarrow & 1 & 2 & 3 \\ \hline & 1 & 2 & 3 & 0 \end{array}$$

$(x-1)(x^2+2x+3) = f(x)$

D. Determine the **two complex roots** of f(x).

Solve $x^2+2x+3=0$

$$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2}$$

$x = -1 \pm i\sqrt{2}$

6. **Simplify** $\sqrt[3]{8b^6} \cdot \sqrt[4]{81c^{16}}$

$2b^2 \cdot 3c^4 = 6b^2c^4$

7. **Solve:** $2 - \frac{1}{x^2+x} = \frac{3}{x+1}$

$x(x+1)$

LCD: $x(x+1)$

$* \; x \neq 0, -1$

$$\frac{2x(x+1)}{\cancel{x(x+1)}} - \frac{1}{\cancel{x(x+1)}} = \frac{3x}{\cancel{x(x+1)}}$$

$2x^2+2x-1 = 3x$

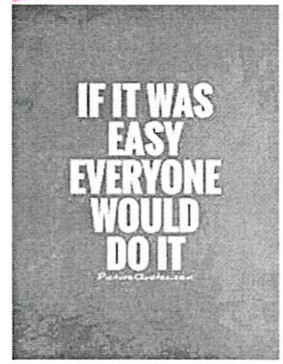
$2x^2-x-1=0$

$$x = \frac{1 \pm \sqrt{(-1)^2-4(2)(-1)}}{2(2)} = \frac{1 \pm 3}{4}$$

$x = \frac{1+3}{4} = 1 \qquad x = \frac{1-3}{4} = \frac{-1}{2}$

$\left\{-\frac{1}{2}, 1\right\}$

8. Use any method of your choice to **prove** that x -5 is a **factor** of $x^4 - 3x^3 - 7x^2 - 11x - 20$.



Factor: x-5

Root: x=5

$(5)^4 - 3(5)^3 - 7(5)^2 - 11(5) - 20 \stackrel{?}{=} 0$

$0 = 0$ ✓ Therefore x-5 is a factor.

9. **Simplify:** $\frac{5x-1}{x^2-3x+2} + \frac{3}{2x-4}$

$(x-2)(x-1) \quad 2(x-2)$

LCD: $2(x-2)(x-1)$

$*x \neq 2, 1$

$$\frac{2(5x-1)}{2(x-2)(x-1)} + \frac{3(x-1)}{2(x-2)(x-1)} = \frac{10x-2+3x-3}{2(x-2)(x-1)}$$

$$= \frac{13x-5}{2(x-2)(x-1)}$$

10. Given the graph shown,

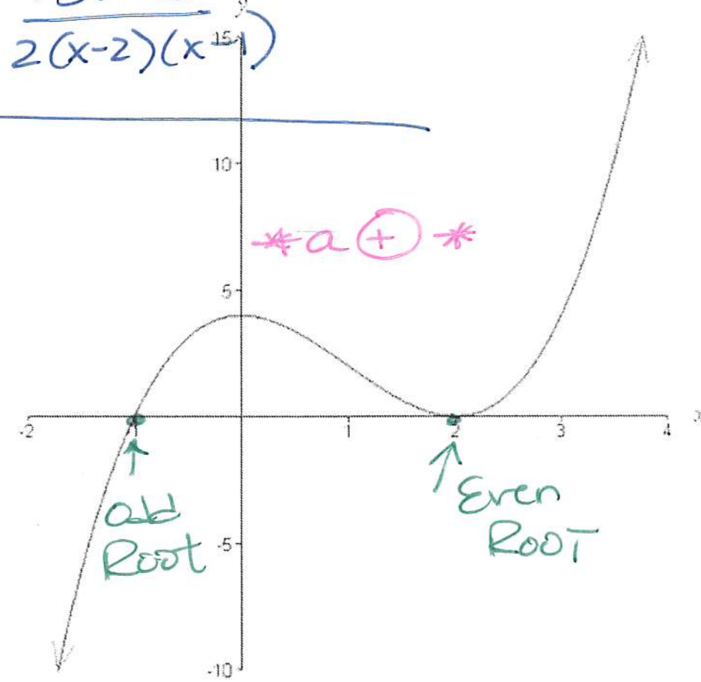
A. State all **roots** of the function.

$x = -1 \quad x = 2 \quad x = 2$

B. State all **factors** of the function.

$(x+1)\ (x-2)\ (x-2)$

C. Write a potential equation for this graph in **standard form**.



$y = (x+1)\boxed{(x-2)(x-2)}$

$y = (x+1)(x^2 - 4x + 4)$

$y = x^3 - 4x^2 + 4x + x^2 - 4x + 4$

$y = x^3 - 3x^2 + 4$

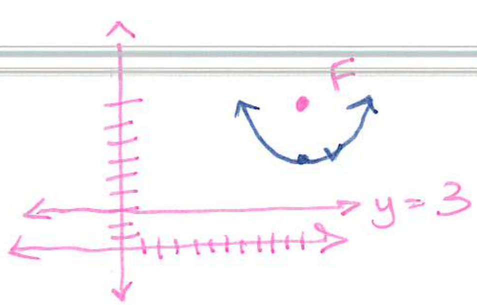
11. **Simplify:** $(3 - xi)(2x^2 i^9)$

$i^9 = i$

$(3 - xi)(2x^2 i) = 6x^2 i - 2x^3 i^2 = 6x^2 i + 2x^3$

$= 2x^3 + 6x^2 i$

12. The **focus** of a given parabola is (11, 9). The **directrix** is given as y = 3. Determine the **equation of this parabola in vertex form.**



Vertex (11, 6)

a (+)

p = 3

$y = \frac{1}{4p}(x-h)^2 + k$

$y = \frac{1}{12}(x-11)^2 + 6$

LCD: $(x-3)(x-4)$

$* x \neq 3, 4$

13. Express in **simplest form:** $\frac{3}{x-3} + \frac{x}{x-4}$

$$\frac{3(x-4)}{(x-3)(x-4)} + \frac{x(x-3)}{(x-3)(x-4)} = \frac{3x-12+x^2-3x}{(x-3)(x-4)} = \frac{x^2-12}{(x-3)(x-4)}$$

14. **Solve** the following system ***(the use of the grid is optional):***

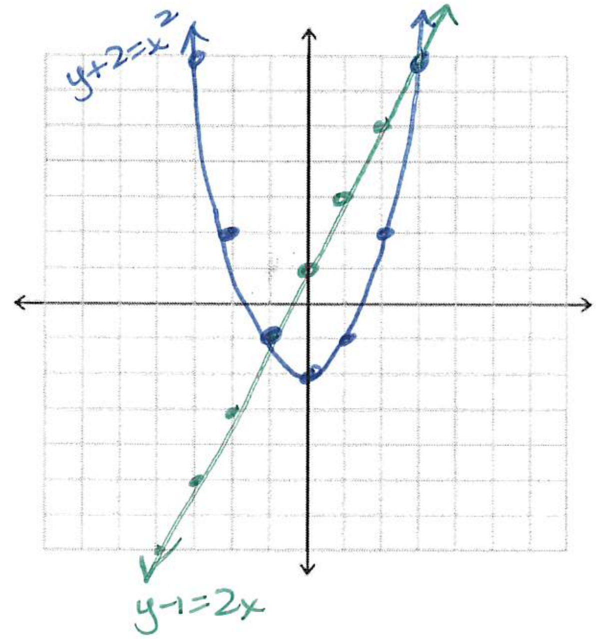
$y - 1 = 2x$

$y + 2 = x^2$

Line: $y = 2x+1$

Parabola $y = x^2 - 2$

$\{(-1,1), (3,7)\}$



15. **Sketch** a graph described below:

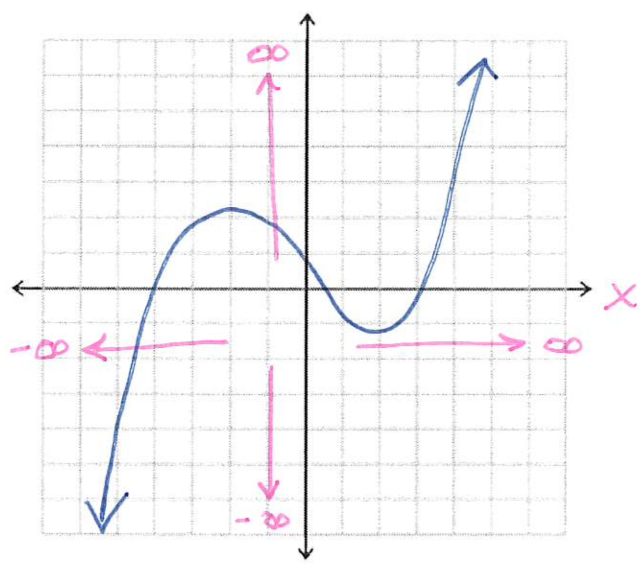
- As $x \to -\infty, f(x) \to -\infty$
- As $x \to \infty, f(x) \to \infty$

A. Is the **degree** of this function even or odd?

Odd

B. Is the value of ***a*** positive or negative?

Positive



16. Solve for x in **simplest radical form:** $x^2 = x - 3$

$x^2 - x + 3 = 0$

$$x = \frac{+1 \pm \sqrt{(-1)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-11}}{2}$$

$$x = \frac{1 \pm i\sqrt{11}}{2}$$

17. **Factor** completely: $4x + 2 - 10x^3 - 5x^2$

$2(2x+1) - 5x^2(2x+1)$

$(2x+1)(2-5x^2)$

18. Determine **the quotient and remainder** for $\frac{2x^4-9x^3+21x^2-26x+10}{2x-3}$.

Q: x^3-3x^2+6x-4

R: -2

$x^3 - 3x^2 + 6x - 4$

$2x-3 \overline{)\,2x^4 - 9x^3 + 21x^2 - 26x + 10}$

$-2x^4 \pm 3x^3$

$-6x^3 + 21x^2$

$+6x^3 \mp 9x^2$

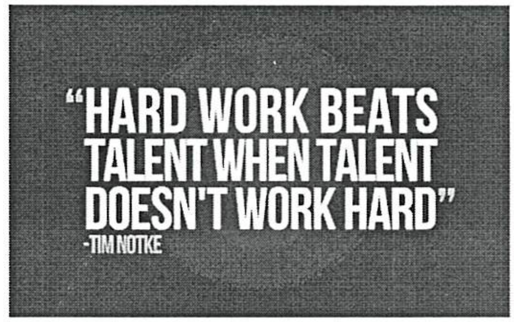
$12x^2 - 26x$

$-12x^2 + 18x$

$-8x + 10$

$+8x \mp 12$

-2



19. **Solve** for x: $\frac{1}{x^2-5x} = \frac{x+7}{x} - 1$

$* x \neq 0, 5$

$x(x-5)$

$\frac{1}{\cancel{x(x-5)}} = \frac{(x+7)(x-5)}{\cancel{x(x-5)}} - \frac{x(x-5)}{\cancel{x(x-5)}}$

$1 = x^2 + 2x - 35 - x^2 + 5x$

$1 = 7x - 35$

$36 = 7x$

$x = \frac{36}{7}$

20. **Simplify completely**: $\frac{u-v}{8v} + \frac{6u-3v}{8v}$

$\frac{u-v+6u-3v}{8v}$

$\frac{7u-4v}{8v}$

21. Use an appropriate procedure to show that x + 3 is **a factor** of $f(x) = x^3 + 2x^2 - 2x + 3$.

Factor: x+3

Root: x=-3

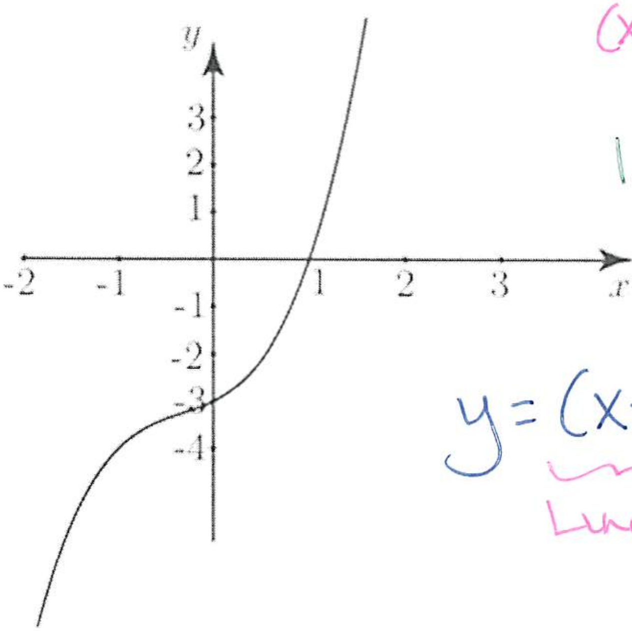
$$\begin{array}{r|rrrr} -3 & 1 & 2 & -2 & 3 \\ & \downarrow & -3 & 3 & -3 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$x^2 - x + 1 \quad r=0$

22. Given the graph of $y = x^3 + x^2 + x - 3$ shown. **Write** the function as a **product of a linear factor and a quadratic factor.**

X=1 Root

(X-1) Factor



$$\begin{array}{r|rrrr} 1 & 1 & 1 & 1 & -3 \\ & \downarrow & 1 & 2 & 3 \\ \hline & 1 & 2 & 3 & 0 \end{array}$$

$y = \underbrace{(x-1)}_{\text{Linear}}\underbrace{(x^2+2x+3)}_{\text{Quadratic}}$