REVIEW OF UNIT 4 (SO FAR...) Key

TOPICS:

- Implicit Differentiation
- Logarithmic Differentiation
- "Weird" Derivative Rules
- Derivatives of Inverse Functions
- Linearizations

1. Find $\frac{dy}{dx}$ of $4xy^2 + 2xy = 6$.

$$4x\left(2y\frac{dy}{dx}\right) + y^2(4) + 2x\left(\frac{dy}{dx}\right) + y(2) = 0$$

$$8xy\frac{dy}{dx} + 4y^2 + 2x\frac{dy}{dx} + 2y = 0$$

$$\frac{dy}{dx}(8xy + 2x) = -2y - 4y^2 \rightarrow \frac{dy}{dx} = \frac{-2y-4y^2}{8xy+2x}$$

2. Find the linearization of $f(x) = \sqrt{x+3}$ at x = 1. Use this linearization to approximate f(0.98).

$$f(1) = 2$$

$$f'(x) = \frac{1}{2}(x+3)^{-\frac{1}{2}}(1)$$

$$f'(1) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$L(x) - 2 = \frac{1}{4}(x-1)$$

$$L(x) = \frac{1}{4}(x-1) + 2$$

$$L(.98) = \frac{1}{4}(.98-1) + 2$$

$$= \frac{1}{4}(-.02) + 2$$

$$= -.005 + 2$$

$$= 1.995$$

3. Find $\frac{dy}{dx}$ if $y = sin^{-1}(4x)$.

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(4x)^2}}(4)$$

$$= \frac{4}{\sqrt{1-16x^2}}$$

4. Find $\frac{dy}{dx}$ if $y = 7^{2x}$.

$$\frac{dy}{dx} = 7^{2x}(\ln 7)(2) \quad \text{OR}$$

$$\ln y = \ln(7^{2x})$$

$$\ln y = 2x(\ln 7)$$

$$\frac{1}{y}\frac{dy}{dx} = 2(\ln 7)$$

$$\frac{dy}{dx} = 2(\ln 7)(7^{2x})$$

5. If $f(x) = x^2, x \geq 0$, find the derivative of $f^{-1}(x)$ at x = 4.

$y = x^2$

$x = y^2$

$\sqrt{x} = f^{-1}(x)$

$$\frac{d f^{-1}(x)}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$\frac{d f^{-1}(4)}{dx} = \frac{1}{4}$$

6. Find $\frac{dy}{dx}$ if $y = log_2(9x)^3$.

$$y = \frac{\ln (9x)^3}{\ln 2}$$

$$y = \frac{3}{\ln 2} \cdot \ln (9x)$$

$$\frac{dy}{dx} = \frac{3}{\ln 2} \cdot \frac{1}{9x} \cdot 9 = \frac{3}{x \ln 2}$$

7. Find $\frac{dy}{dx}$ if $y = -8x^{2x}$.

$y = -8(x)^{2x}$

$\ln y = \ln(-8(x)^{2x})$

$\ln y = \ln(-8) + 2x \ln(x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = 0 + 2x\left(\frac{1}{x}\right) + \ln(x)(2)$

$\frac{1}{y}\frac{dy}{dx} = 2 + 2\ln x$

$\frac{dy}{dx} = (2 + 2\ln x)(-8x^{2x})$

8. Find $\frac{dy}{dx}$ if $y = log_5\left(\frac{x}{x-1}\right)$.

$$y = \frac{\ln\left(\frac{x}{x-1}\right)}{\ln 5}$$

$$\frac{dy}{dx} = \frac{1}{\ln 5}\left(\frac{1}{\frac{x}{x-1}}\right)\left(\frac{(x-1)(1) - x(1)}{(x-1)^2}\right)$$

$$= \frac{1}{\ln 5}\left(\frac{\cancel{x-1}}{x}\right)\left(\frac{-1}{(x-1)^{\cancel{2}}}\right)$$

$$= \frac{-1}{(\ln 5)(x)(x-1)}$$