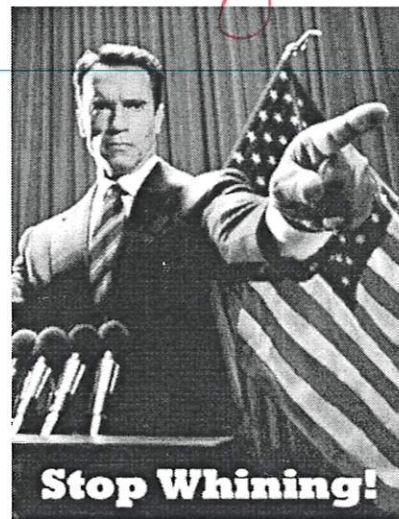


# REVIEW OF UNIT 4 (SO FAR...)

Key

## TOPICS:

- Implicit Differentiation
- Logarithmic Differentiation
- "Weird" Derivative Rules
- Derivatives of Inverse Functions
- Linearizations



1. Find  $\frac{dy}{dx}$  of  $4xy^2 + 2xy = 6$ .

$$4x \left( 2y \frac{dy}{dx} \right) + y^2 (4) + 2x \left( \frac{dy}{dx} \right) + y(2) = 0$$

$$8xy \frac{dy}{dx} + 4y^2 + 2x \frac{dy}{dx} + 2y = 0$$

$$\frac{dy}{dx} (8xy + 2x) = -2y - 4y^2$$

$$\frac{dy}{dx} = \frac{-2y - 4y^2}{8xy + 2x}$$

2. Find the linearization of  $f(x) = \sqrt{x+3}$  at  $x=1$ . Use this linearization to approximate  $f(0.98)$ .

$$f(1) = 2$$

$$f'(x) = \frac{1}{2}(x+3)^{-\frac{1}{2}} (1)$$

$$f'(1) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$L(x) - 2 = \frac{1}{4}(x-1)$$

$$L(x) = \frac{1}{4}(x-1) + 2$$

$$L(0.98) = \frac{1}{4}(0.98-1) + 2$$

$$= \frac{1}{4}(-0.02) + 2$$

$$= -0.005 + 2$$

$$= 1.995$$

3. Find  $\frac{dy}{dx}$  if  $y = \sin^{-1}(4x)$ .

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(4x)^2}} (4)$$

$$= \frac{4}{\sqrt{1-16x^2}}$$

4. Find  $\frac{dy}{dx}$  if  $y = 7^{2x}$ .

$$\frac{dy}{dx} = 7^{2x} (\ln 7) (2) \quad \text{OR}$$

$$\ln y = \ln(7^{2x})$$

$$\ln y = 2x (\ln 7)$$

$$\frac{1}{y} \frac{dy}{dx} = 2 (\ln 7)$$

$$\frac{dy}{dx} = 2 (\ln 7) (7^{2x})$$

5. If  $f(x) = x^2, x \geq 0$ , find the derivative of  $f^{-1}(x)$  at  $x = 4$ .

$$y = x^2$$

$$x = y^2$$

$$\sqrt{x} = f^{-1}(x)$$

$$\frac{d f^{-1}(x)}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$

6. Find  $\frac{dy}{dx}$  if  $y = \log_2(9x)^3$ .

$$y = \frac{\ln(9x)^3}{\ln 2}$$

$$\frac{d f^{-1}(4)}{dx} = \frac{1}{4}$$

$$y = \frac{3}{\ln 2} \cdot \ln(9x)$$

$$\frac{dy}{dx} = \frac{3}{\ln 2} \cdot \frac{1}{9x} \cdot 9 = \frac{3}{x \ln 2}$$

7. Find  $\frac{dy}{dx}$  if  $y = -8x^{2x}$ .

$$y = -8(x)^{2x}$$

$$\ln y = \ln(-8(x)^{2x})$$

$$\ln y = \ln(-8) + 2x \ln(x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 0 + 2x \left(\frac{1}{x}\right) + \ln(x) (2)$$

$$\frac{1}{y} \frac{dy}{dx} = 2 + 2 \ln x$$

$$\frac{dy}{dx} = (2 + 2 \ln x) (-8x^{2x})$$

8. Find  $\frac{dy}{dx}$  if  $y = \log_5\left(\frac{x}{x-1}\right)$ .

$$y = \frac{\ln\left(\frac{x}{x-1}\right)}{\ln 5}$$

$$\frac{dy}{dx} = \frac{1}{\ln 5} \left( \frac{1}{\frac{x}{x-1}} \right) \left( \frac{(x-1)(1) - x(1)}{(x-1)^2} \right)$$

$$= \frac{1}{\ln 5} \left( \frac{x-1}{x} \right) \left( \frac{-1}{(x-1)^2} \right)$$

$$= \frac{-1}{(\ln 5)(x)(x-1)}$$

