Unit 5 - Polynomials - Study Guide

Important Vocabulary

Term – A number, variable or product of numbers and variables
Like/similar terms – Two or more terms that have the same variable(s) with the same exponents
Unlike terms – Two or more terms with different variables and/or different powers
Monomial – A single variable or number, or a product of a coefficient and one or more variables with exponents that are whole numbers
Binomial – A polynomial with two terms
Trinomial – A polynomial with three terms
Simplest form – When a polynomial contains no like terms
Standard form – When a polynomial is written with the exponents in descending order
Coefficient – The number being multiplied to the variable (written in front of variable)
Leading coefficient – The coefficient of the variable with the highest degree
Constant – A number that is not attached to any variable
Variable – A letter or symbol used to replace a number
Degree of a polynomial – the degree of the monomial with the greatest exponent

Laws of Exponents

- Anything to the FIRST power is ITSELF
  $20^1 = 20$
- Anything to the ZERO power is ONE
  $20^0 = 1$
- When MULTIPLYING exponents that have the same base, ADD the powers
  $x^2 \bullet x^4 = x^6$
- When DIVIDING exponents that have the same base, SUBTRACT the powers
  $x^{10} \div x^8 = x^2$
- When raising an exponent to another power, MULTIPLY the powers.
  $(x^2)^4 = x^8$

Polynomial

An expression made up of variables and constants.
The exponents are always WHOLE NUMBERS!!!

Adding/Subtracting Polynomials

1. Distribute
2. Combine like terms
3. Write your answer in standard form

Tips:

- Powers NEVER change
- Write in coefficients of 1
  EX: $2x + x = 2x + 1x = 3x$, not $2x$
- Be careful when distributing a negative
  EX: $(3x + 1) - (2x - 4) = 3x + 1 - 2x + 4$
- Subtract x from y looks like $y - x$
- SUM or TOTAL means ADD
- DIFFERENCE means SUBTRACT
- PERIMETER means add up ALL of the sides

Multiplying Polynomials

1. Write in exponents and coefficients of 1
2. Distribute (if necessary)
3. Multiply coefficients
4. Multiply variables by ADDING powers
   (or if power to a power MULTIPLY)

Tips:

- Use the LAWS OF EXPONENTS
- Powers ALMOST ALWAYS change
- PRODUCT means MULTIPLY
- Be careful of integers
- Pay attention to what type of polynomials you start with (never end with more terms than your largest polynomial)
  EX: $(4x^4y^3z^3)(2x^2y^2z) = 8x^6y^5z^4$
  (mono*mono = mono)
- Expand whenever you see a power
  EX: $(5x)^3 = 5x \bullet 5x \bullet 5x = 125x^3$
  EX: $(x + 2)^2 = (x + 2)(x + 2)$
- AREA and VOLUME means multiply

Adding vs. Multiplying

Simplify: $3x^2 + 5x^2$	Simplify: $3x^2 \bullet 5x^2$
$= 8x^2$	$= 15x^4$
Add coefficients	Multiply coefficients
Powers don't change	Add powers

Double Distributing (and beyond)

Be careful of INTEGER RULES!

(binomial)(binomial)	(binomial)^2	(binomial)(trinomial)
$(x+3)(x-7)$	$(x-5)^2$	$(x-1)(x^2+3x-2)$
$x(x-7)+3(x-7)$ $x^2-7x+3x-21$ $x^2-4x-21$	$(x-5)(x-5)$ $x(x-5)-5(x-5)$ $x^2-5x-5x+25$ $x^2-10x+25$	$x(x^2+3x-2)-1(x^2+3x-2)$ $x^3+3x^2-2x-1x^2-3x+2$ x^3+2x^2-5x+2

Completing the Square

Example:

$x^2-8x+22$

$x^2-8x+22$

$(x-4)(x-4)+22-16$

$-4 \times -4 = 16$

$(x-4)^2+6$

$x^2-8x+22$
is the same as
$(x-4)^2+6$

Closure

A set is CLOSED under an operation if it produces a member of that same set. Polynomials are closed under addition, subtraction, multiplication but NOT division.

EXAMPLES:

Addition: $3x^2+8x^2=11x^2$

Subtraction:

$$3x^2-8x^2=-5x^2$$

Multiplication:

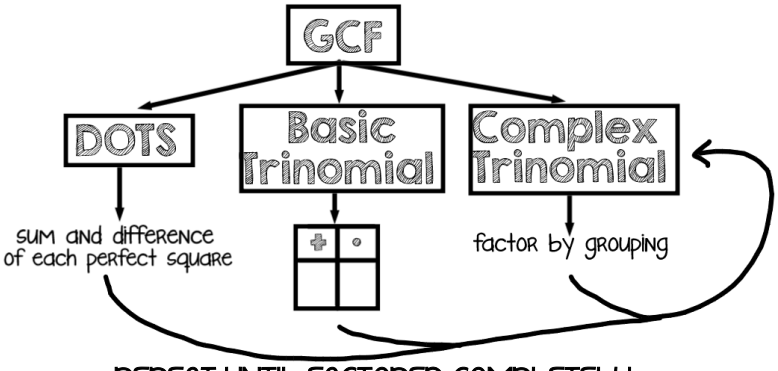
$$3x^2 \bullet 8x^2=24x^4$$

NON-EXAMPLE:

Division: $\frac{10x^3}{5x^8}=2x^{-5}$ ***NOT a polynomial

Factoring Completely

- Remember: we always RE-WRITE and NEVER change the value of an expression
- Always check by multiplying back through to make sure you get what you started with (or use calculator magic number technique)



Examples:

$x^2+4x-45$

S|P
4 | -45

$9+-5=4$ ✓
$9 \cdot -5=-45$ ✓

$(x+9)(x-5)$

x^4+7x^2+10

S|P
7 | 10

$5+2=7$ ✓
$5 \cdot 2=10$ ✓

$(x^2+5)(x^2+2)$

b^2-25

DOTS
$b \cdot b=b^2$ ✓
$5 \cdot 5=25$ ✓
subtract ✓

$(b+5)(b-5)$

$2m^3-26m^2+72m$

GCF: $2m$

S|P
-13 | 36

$-9+-4=-13$ ✓
$-9 \cdot -4=36$ ✓

$2m(m^2-13m+36)$
$2m(m-9)(m-4)$

GCF: 3

S|P
-11 | 18

$-9 \cdot -2=18$ ✓
$-9+-2=-11$ ✓

$18a^2-33a+9$
$3(6a^2-11a+3)$
$3(6a^2-9a-2a+3)$
$3[3a(2a-3)-1(2a-3)]$
$3(3a-1)(2a-3)$

x^4-81

DOTS
$x^2 \cdot x^2=x^4$ ✓
$9 \cdot 9=81$ ✓

DOTS
$x \cdot x=x^2$ ✓
$3 \cdot 3=9$ ✓

$(x^2+9)(x^2-9)$
$(x^2+9)(x+3)(x-3)$