Unit 6 - Quadratic Functions - Study Guide

Important Vocabulary

standard form: $y = ax^2 + bx + c$
leading coefficient: the "a" in standard form
roots/zeros/x-intercepts/solutions: the x-values of the x-intercepts, **when y = 0**
vertex: the **turning point** of the parabola: max/min
y-intercept: the point where the graph crosses the y-axis, **when x = 0**
axis of symmetry: the **x-value** of the **vertex**; x =
maximum: the **highest** y-value
minimum: the **lowest** y-value
concave up: vertex is a **minimum**
concave down: vertex is a **maximum**

Graphing on the TI-Nspire

In Graphing Scratchpad...

- ADD A FUNCTION
  Press TAB, then ↑ or ↓ to change between functions
- CHANGE WINDOW
  Pick #'s: MENU → 4 → 1
  ZOOM FIT: MENU → 4 → A

Simplifying Radicals

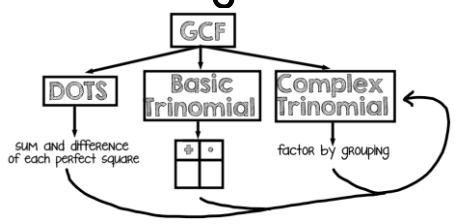
1. Find the BIGGEST perfect square factor
2. Write as the product of 2 radicals (make sure the perfect square is first)
3. Evaluate the perfect square

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

EXAMPLE: Simplify $x = 3\sqrt{48}$

$$x = 3 \bullet \sqrt{16} \bullet \sqrt{3}$$
$$x = 3 \bullet 4 \bullet \sqrt{3}$$
$$x = 12\sqrt{3}$$

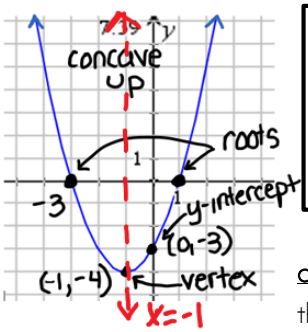
Factoring REVIEW



REPEAT UNTIL FACTORED COMPLETELY

Different Forms of a Parabola

FORM	TELLS US	EXAMPLE
Standard Form	ax^2 • Opens: UP: a is + DOWN: a is − • NARROW: $a > 1$ WIDE: $0 < a < 1$ • y-intercept: is the CONSTANT	$y = x^2 + 2x - 3$ ⇨ opens up b/c x^2 is positive ⇨ y-intercept at $(0, -3)$
Vertex Form	$a(x-h)^2 + k$ • Opens: UP: a is + DOWN: a is − • NARROW: $a > 1$ WIDE: $0 < a < 1$ • Vertex: $(-h, k)$ h is always OPPOSITE SIGN	$y = (x+1)^2 - 4$ ⇨ opens up b/c number in front of parenthesis is positive ⇨ vertex: $(-1, -4)$
Factored Form	• roots/zeros set each factor =0	$y = (x-1)(x+3)$ ⇨ roots at x = 1 and x = −3



The SAME function written 3 different ways!
$y = x^2 + 2x - 3$
$y = (x+1)^2 - 4$
$y = (x-1)(x+3)$

axis of symmetry: $x = -\frac{b}{2a}$
the x-value of the vertex: x =

Completing the Square

- GOAL: Write in vertex form (or solve...found on back)
- Just re-writing: it should always be the SAME equation from start to end

EXAMPLE #1

$y = x^2 - 8x + 22$

$y = x^2 - 8x + 22$
$y = x^2 - 8x + 16 - 16 + 22$
$y = (x-4)(x-4) - 16 + 22$
$y = (x-4)^2 + 6$

$y = x^2 - 8x + 22$
is the same as
$y = (x-4)^2 + 6$

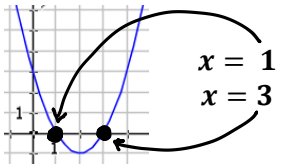
EXAMPLE #2

$0 = -2x^2 + 20x - 61$

$0 = -2x^2 + 20x - 61$
$0 = -2(x^2 - 10x + 25) + 50 - 61$
$0 = -2(x-5)(x-5) - 11$
$0 = -2(x-5)^2 - 11$

$0 = -2x^2 + 20x - 61$
is the same as
$0 = -2(x-5)^2 - 11$

Solving Quadratic Equations ~ Finding Roots/Zeros

	Steps	Examples
Graphically	Graph to find the x-intercepts/roots ***Using the TI-Nspire: MENU→6→1 Do this for EVERY root	$y = x^2 - 4x + 3$  $x = 1$ $x = 3$
Factoring	1. Factor 2. Set each factor = 0 3. Solve to find roots	$y = x^2 + 7x - 18$ SET = 0 !!! $0 = x^2 + 7x - 18$ $0 = (x + 9)(x - 2)$ $0 = x + 9 \quad 0 = x - 2$ $\boldsymbol{x = -9 \quad x = 2}$
Inverse Operations	1. Move constant to other side 2. Take square root of both sides ($\pm$!!!!) ***use when there isn't a "bx" term	$y = x^2 - 25$ SET = 0 !!! $0 = x^2 - 25$ $25 = x^2$ $x = \sqrt{25}$ $x = \pm 5$ so... $\boldsymbol{x = -5}$ and $\boldsymbol{x = 5}$
Completing the Square	1. Write in vertex form 2. Move constant to other side 3. Take square root of both sides 4. Spit into 2 equations 5. Solve ***use only if you CAN'T factor & when "b" is EVEN	$y = x^2 + 6x - 1$ SET = 0 !!! $0 = x^2 + 6x - 1$ $0 = x^2 + 6x + 9 - 9 - 1$ $0 = (x + 3)^2 - 10$ $10 = (x + 3)^2$ $\pm\sqrt{10} = x + 3$ $x + 3 = +\sqrt{10} \quad x + 3 = -\sqrt{10}$ $\boldsymbol{x = -3 + \sqrt{10} \quad x = -3 - \sqrt{10}}$ (in simplest radical form)
Quadratic Formula	1. Identify a, b and c 2. Substitute and solve ***use only if you CAN'T factor and when "b" is ODD ***Using the TI-Nspire: MENU→3→3	$y = x^2 + 4x - 3$ SET = 0 !!! $0 = x^2 + 4x - 3$ a = 1 b = 4 c = -3 $x = -\frac{4}{2(1)} \pm \frac{\sqrt{4^2 - 4(1)(-3)}}{2(1)}$ $x = -2 + \frac{\sqrt{28}}{2} \quad x = -2 - \frac{\sqrt{28}}{2}$ $\boldsymbol{x = 0.6 \quad x = -4.6}$ (rounded to the nearest tenth)

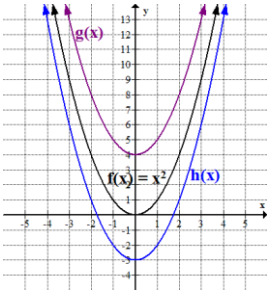
Transforming Parabolas

OUTSIDE parenthesis (constant):

UP + or DOWN -

$g(x) = x^2 + 4$ up 4

$h(x) = x^2 - 3$ down 3



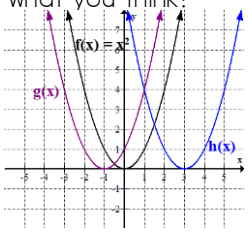
INSIDE parenthesis

LEFT + or RIGHT -

** opposite of what you think!**

$g(x) = (x + 1)^2$ left 1

$h(x) = (x - 3)^2$ right 3



IN FRONT

reflects over the x-axis if -

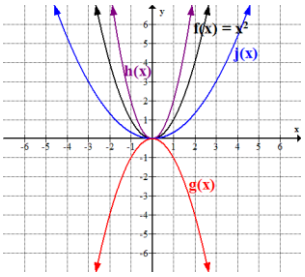
wider if a fraction smaller than 1

narrower if bigger than 1

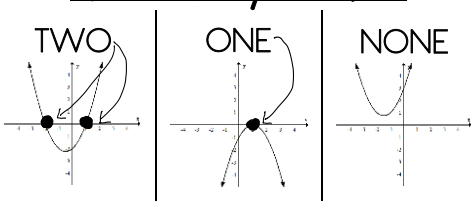
$g(x) = -x^2$ reflects over x-axis

$h(x) = 2x^2$ narrower

$j(x) = \frac{1}{3}x^2$ wider



How many roots?



Solving: Linear versus Quadratic

Linear: get x alone	Quadratic: set = 0
$5x - 8 + 3x = 6(x - 1)$ $5x - 8 + 3x = 6x - 6$ $5x + 3x - 6x = 8 - 6$ $\frac{-1x}{-1} = \frac{2}{-1}$ $\boxed{x = -2}$	$x^2 - 7 = x + 5$ $x^2 - x - 7 - 5 = 0$ $x^2 - x - 12 = 0$ $(x - 4)(x + 3) = 0$ $x - 4 = 0 \quad \boxed{x = 4}$ $x + 3 = 0 \quad \boxed{x = -3}$

Quadratic Formula

In the quadratic $0 = ax^2 + bx + c$, the roots can be found using...

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Quadratic Word Problems Tips: always define variables and make sure the answer makes sense!